UMC Utrecht

Model-based reconstruction methods for MRI

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Magnetic Resonance Imaging

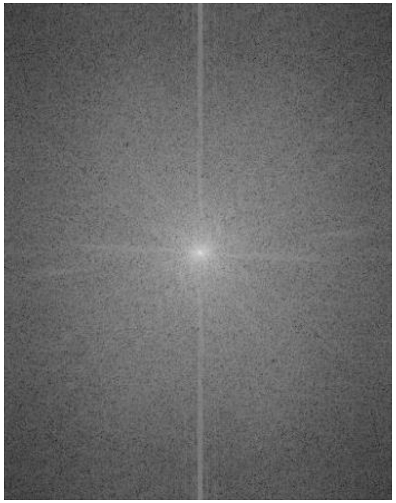
Bloch Equations

$$s(t) = \int_{volume} m(\boldsymbol{x}, t)\, d\boldsymbol{x}$$

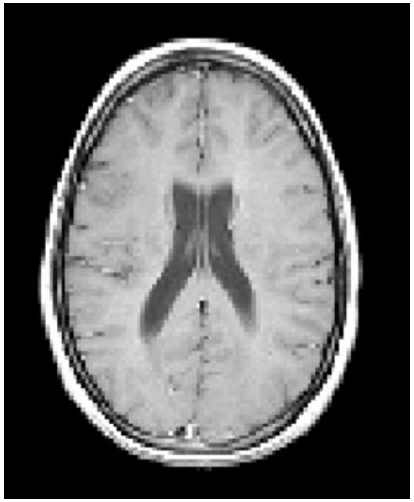
$$\dot{m}(\boldsymbol{x}, t) = F(\, m(\boldsymbol{x}, t), \rho(\boldsymbol{x}), T_1(\boldsymbol{x}), T_2(\boldsymbol{x}), B_1(\boldsymbol{x}), B_0(\boldsymbol{x}), \ldots\,)$$

Fourier Spatial Reconstruction

$$s(t) = \int_{volume} m(\boldsymbol{x}, t)\, e^{i\gamma \boldsymbol{G}\boldsymbol{x}} d\boldsymbol{x}$$



Inverse FFT



Magnetic Moment

$m(\boldsymbol{x}, t_1)$

Magnetic Moment

$m(\boldsymbol{x}, t_1)$

$m(\boldsymbol{x}, t_2)$

Magnetic Moment

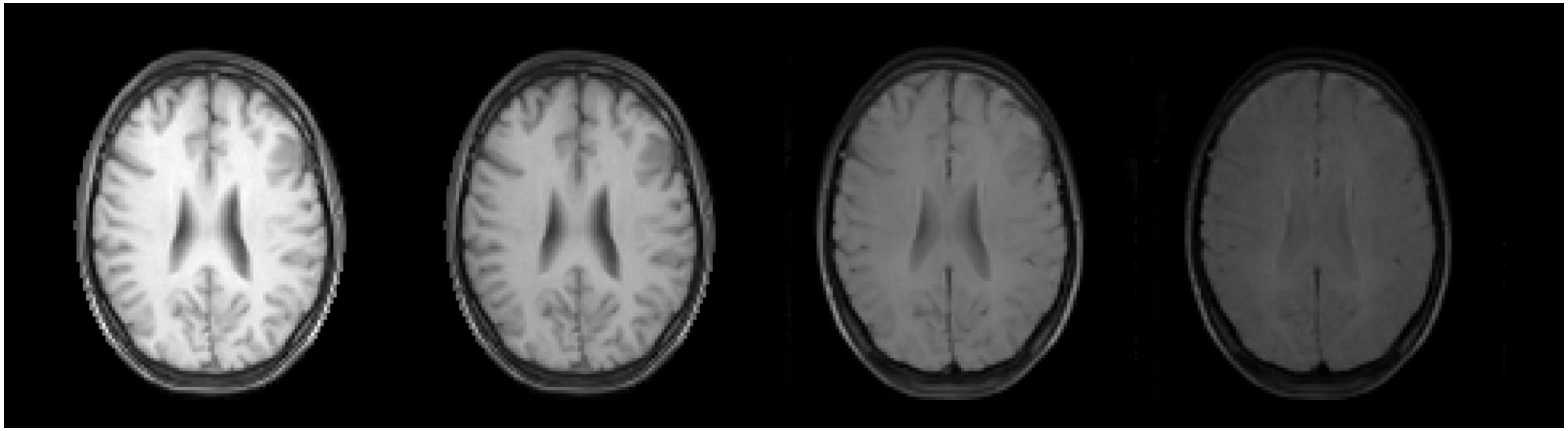
$m(\boldsymbol{x}, t_1)$

$m(\boldsymbol{x}, t_2)$

$\longrightarrow \quad \rho, T_1, T_2$

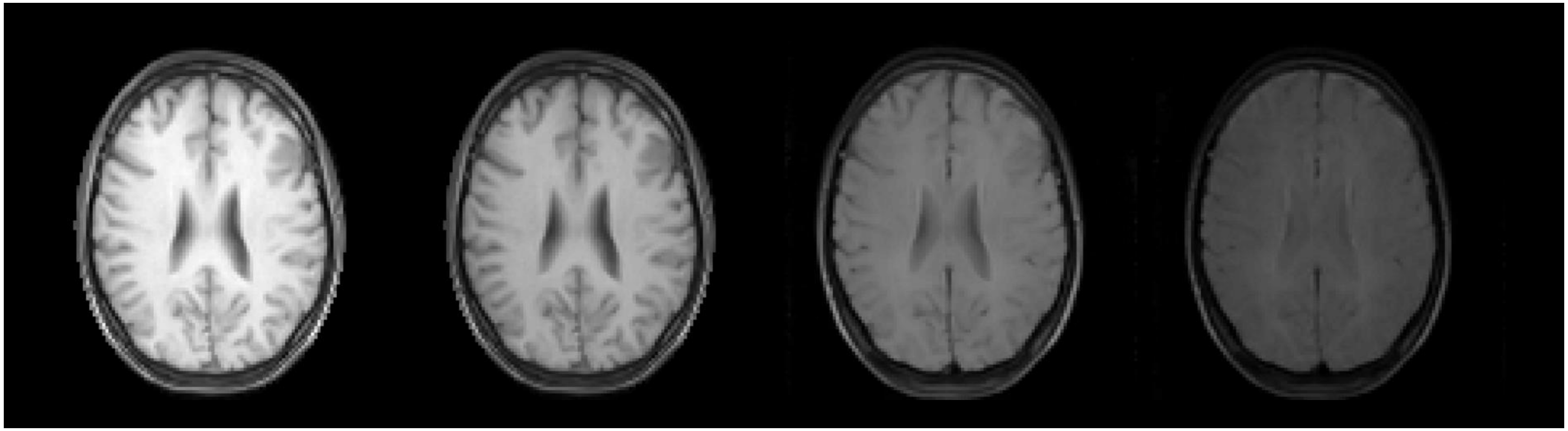
Time Evolution

$$\dot{m}(t) = F(\, m(t), \rho, T_1, T_2, B_1, B_0, \ldots\,)$$



Simplified equations

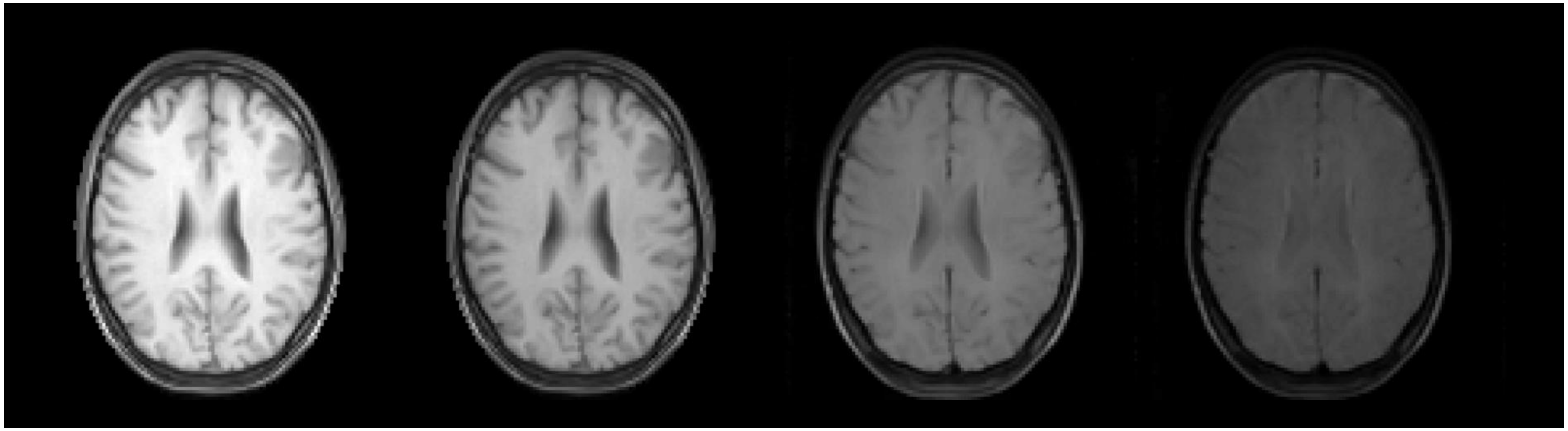
$$\dot{m}(t) = F(\, m(t), \rho, T_1, T_2, B_1, B_0, \dots\,)$$



$$m(t) \sim \rho\, e^{-t/T_2}$$

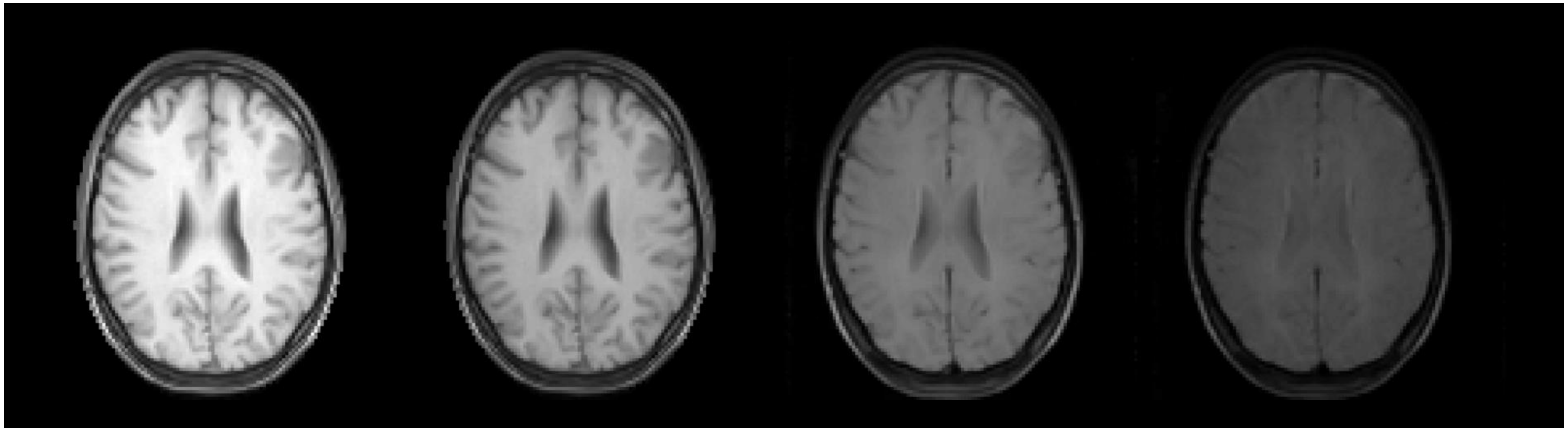
Minimization problem

$$\rho, T_1, T_2, B_1, B_0 = \min \; m(t) - F(t, \rho, T_1, T_2, B_1, B_0)$$

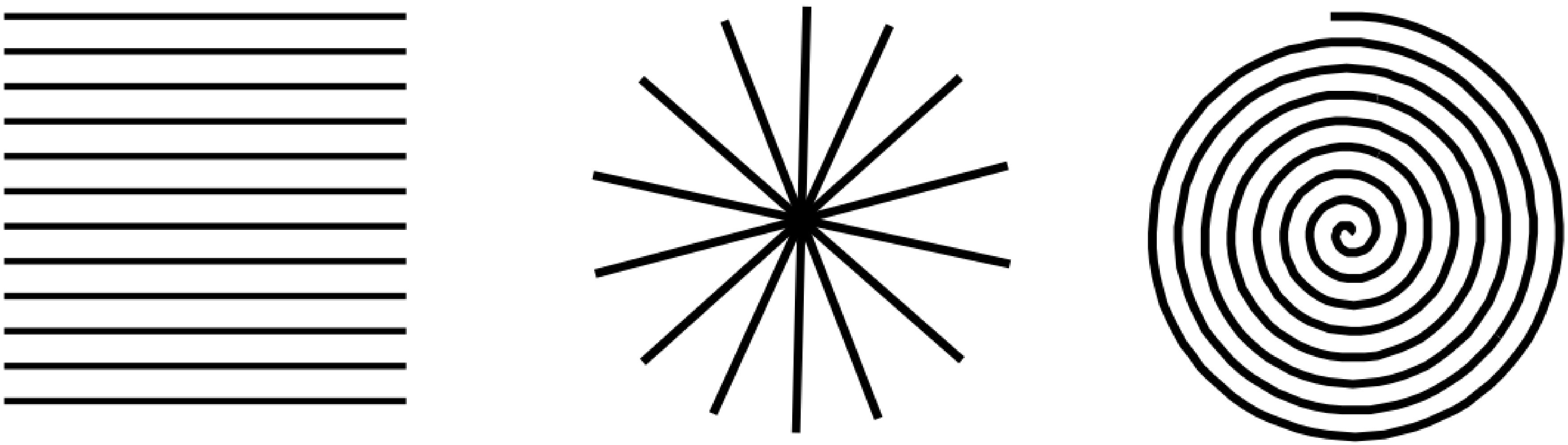


Discrete Time Map

$$m_{t+1} = F(\, m_t, \rho, T_1, T_2, B_1, B_0, \ldots)$$

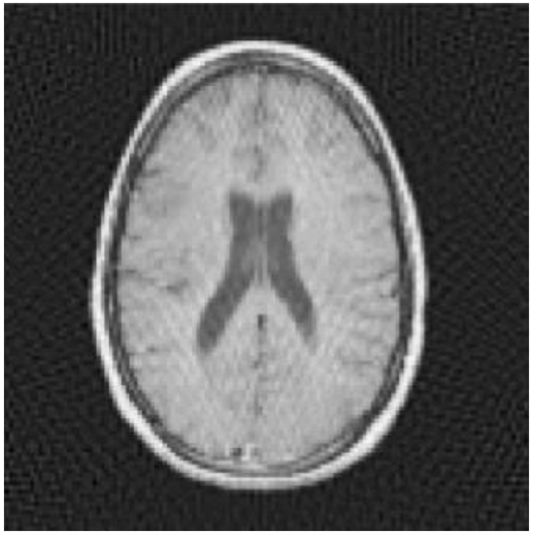


Undersampling

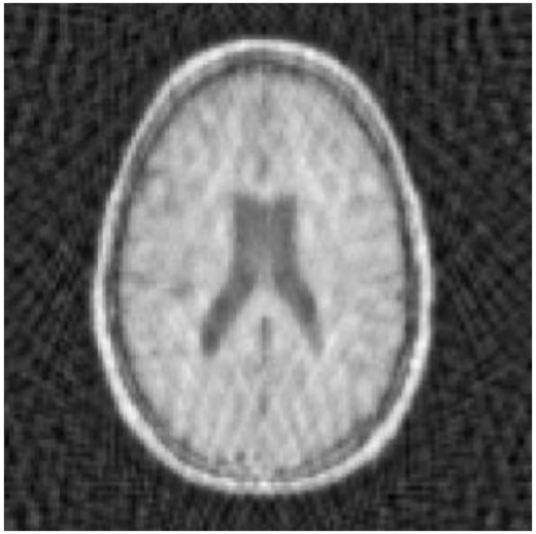


Radial undersampling

1 x

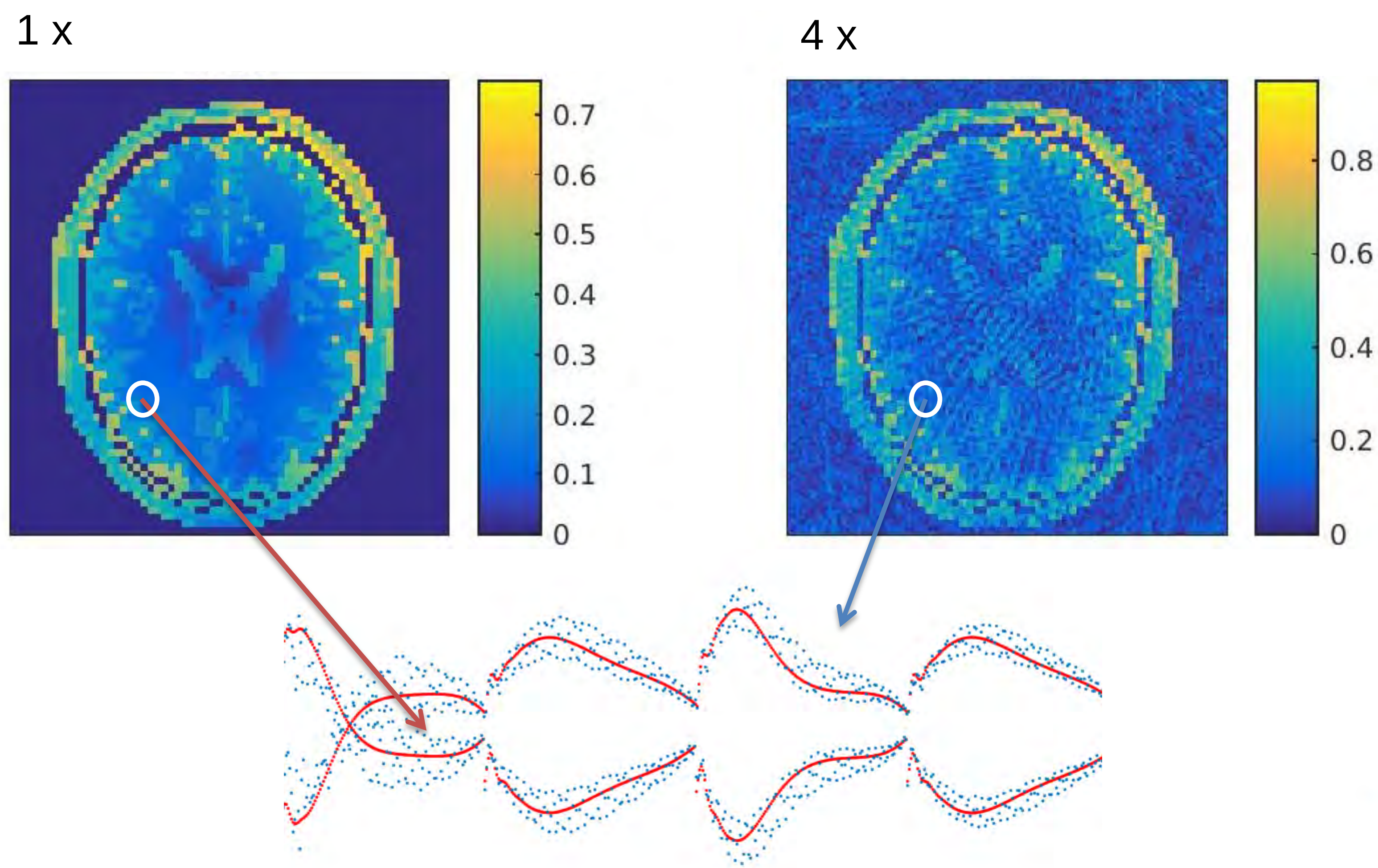


2 x

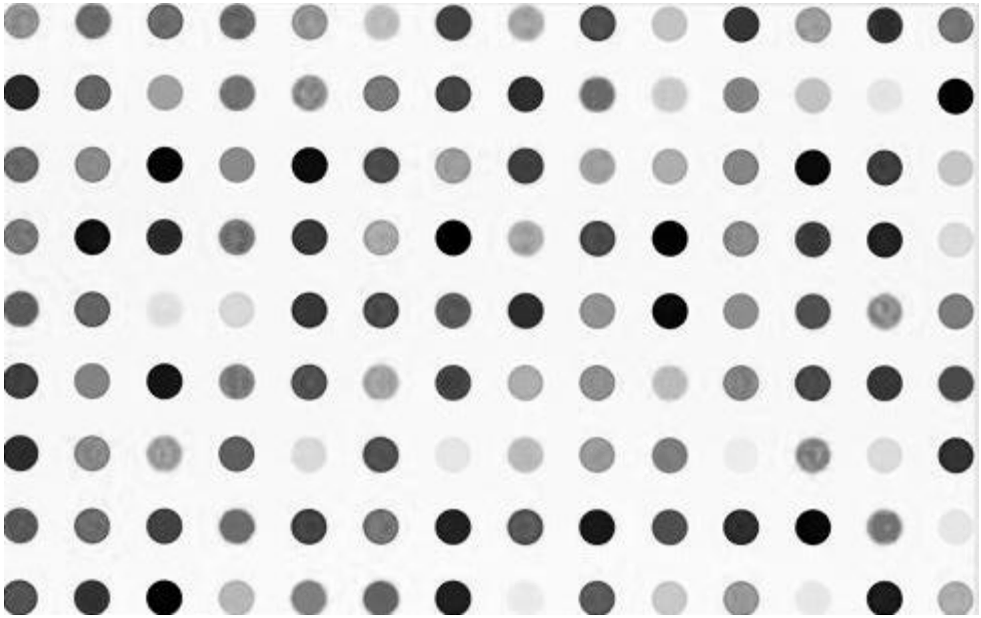


4 x

Artifacts =? Pseudo-noise



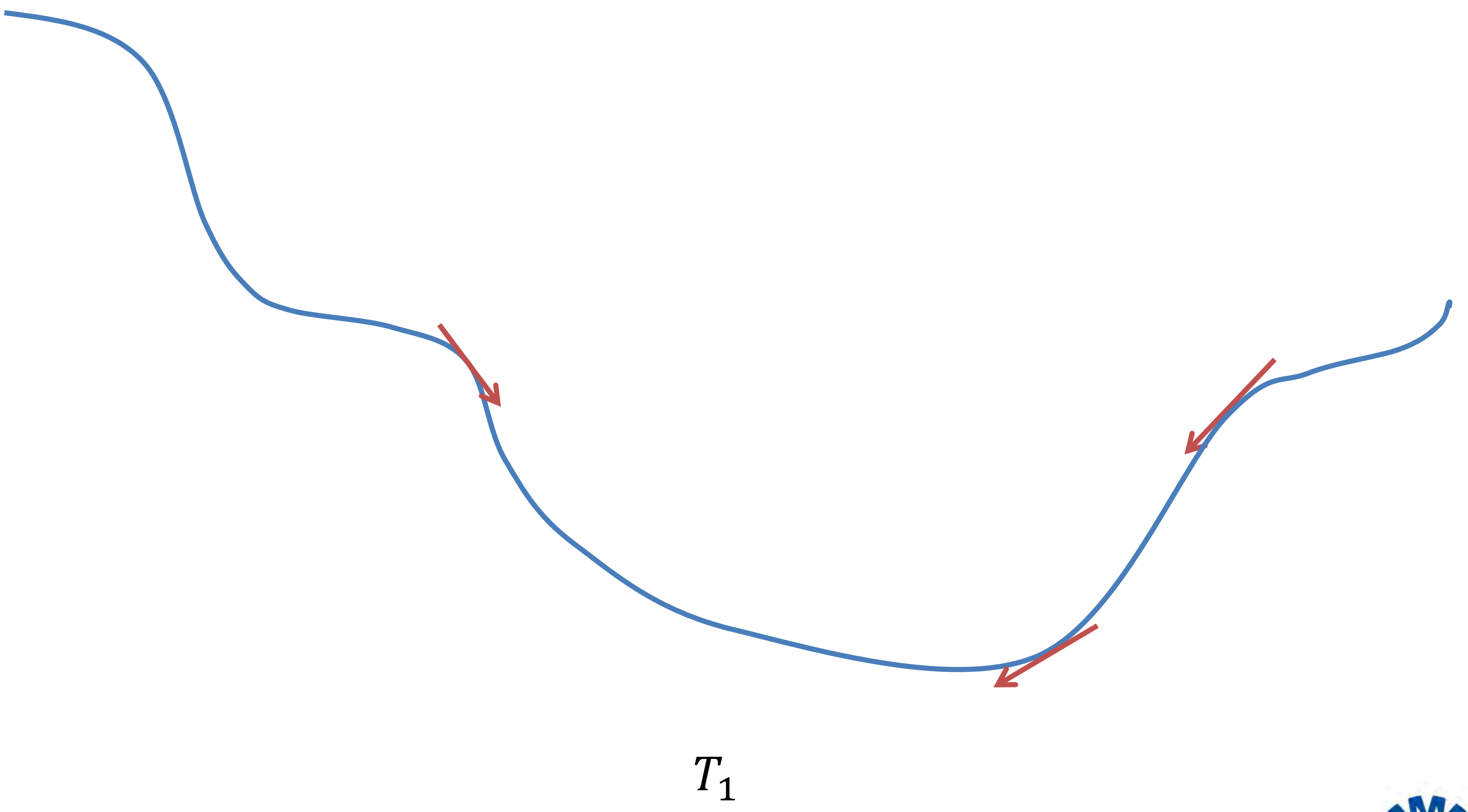
Dictionary matching



T_2

T_1

Gradient-based iterations



Kalman Filter

$$m_{t+1} = F(\, m_t, \alpha, \dots\,)$$

$$s_{t+1} = G(m_{t+1})$$

Dual Kalman Filter

$$m_{t+1} = F(m_t, \alpha, \ldots) \qquad \alpha_{t+1} = \alpha_t$$

$$s_{t+1} = G(m_{t+1})$$

Predict Covariances

$$m_{t+1} \quad = F(\, m_t, \alpha, \ldots\,)$$

$$\alpha_{t+1} \quad = \alpha_t$$

$$P_{m,t+1} = \frac{\partial F}{\partial m} P_{m,t} \left(\frac{\partial F}{\partial m}\right)^T + R_m$$

$$P_{\alpha,t+1} = P_{\alpha,t} + R_\alpha$$

$$s_{t+1} \quad = G(m_{t+1})$$

Compute the residual

$$r_{t+1} = d_{t+1} - s_{t+1}$$

$$P_{r,m,t+1} = \frac{\partial G}{\partial m} P_{m,t+1} \left(\frac{\partial G}{\partial m}\right)^T + R_s$$

$$P_{r,\alpha,t+1} = \frac{\partial G}{\partial \alpha} P_{\alpha,t+1} \left(\frac{\partial G}{\partial \alpha}\right)^T + R_s$$

Correction of estimates

$$K_{m,t+1} = P_{m,t+1} \left(\frac{\partial G}{\partial m}\right)^T \left(P_{r,m,t+1}\right)^{-1}$$

$$m_{t+1} = m_{t+1} + K_{m,t+1} r_{t+1}$$

$$P_{m,t+1} = \left(I + K_{m,t+1} \frac{\partial G}{\partial m}\right) P_{m,t+1}$$

Correction of estimates

$$K_{m,t+1} = P_{m,t+1}\left(\frac{\partial G}{\partial m}\right)^T \left(P_{r,m,t+1}\right)^{-1}$$

$$K_{\alpha,t+1} = P_{\alpha,t+1}\left(\frac{\partial G}{\partial \alpha}\right)^T \left(P_{r,\alpha,t+1}\right)^{-1}$$

$$m_{t+1} = m_{t+1} + K_{m,t+1} r_{t+1}$$

$$\alpha_{t+1} = \alpha_{t+1} + K_{\alpha,t+1} r_{t+1}$$

$$P_{m,t+1} = \left(I + K_{m,t+1}\frac{\partial G}{\partial m}\right) P_{m,t+1}$$

$$P_{\alpha,t+1} = \left(I + K_{\alpha,t+1}\frac{\partial G}{\partial \alpha}\right) P_{\alpha,t+1}$$

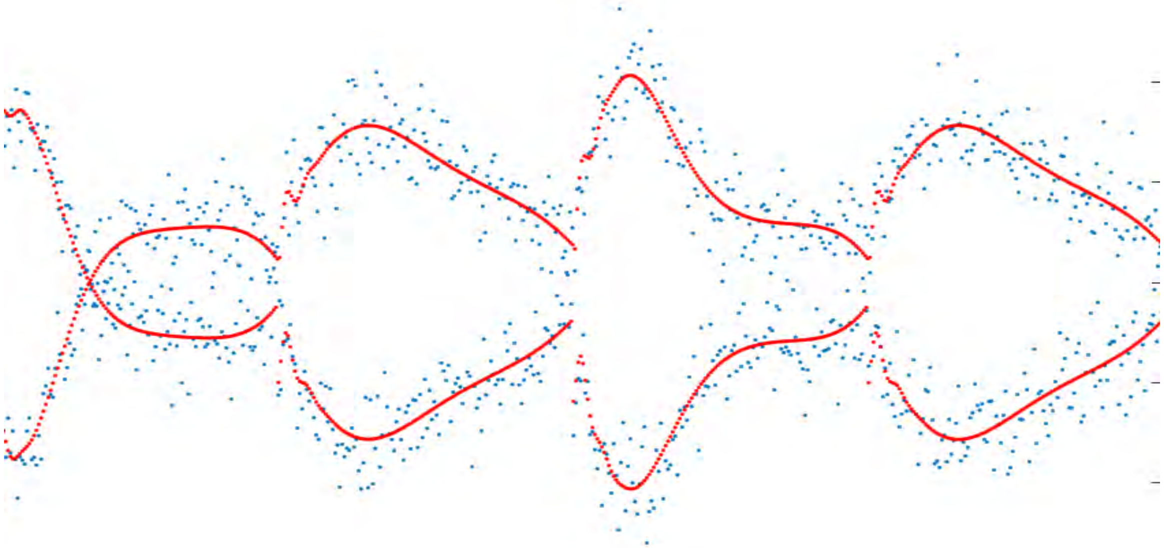
Settings!

R_m, R_α, R_s Noise covariances

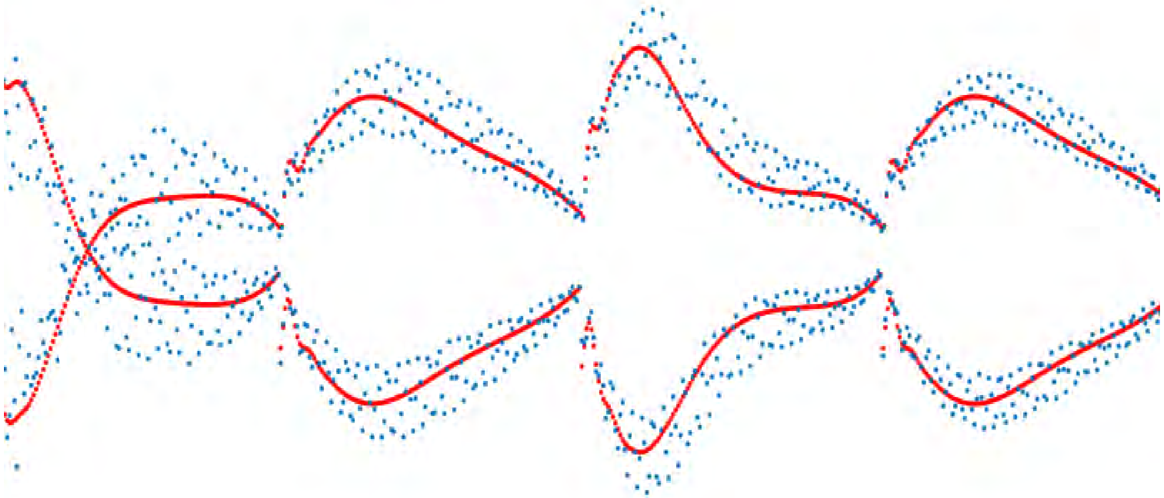
Time stepsize?

Recycling the data

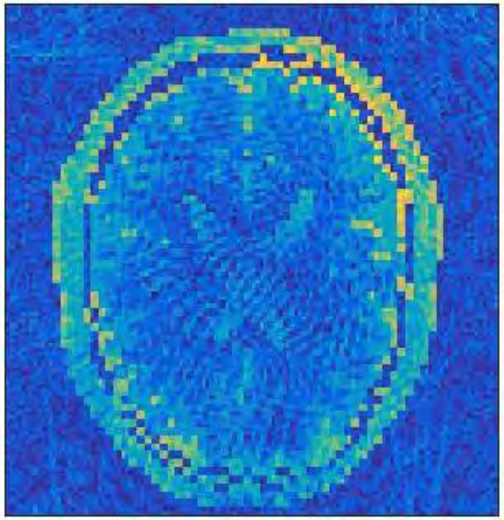
Noise vs Pseudo-noise



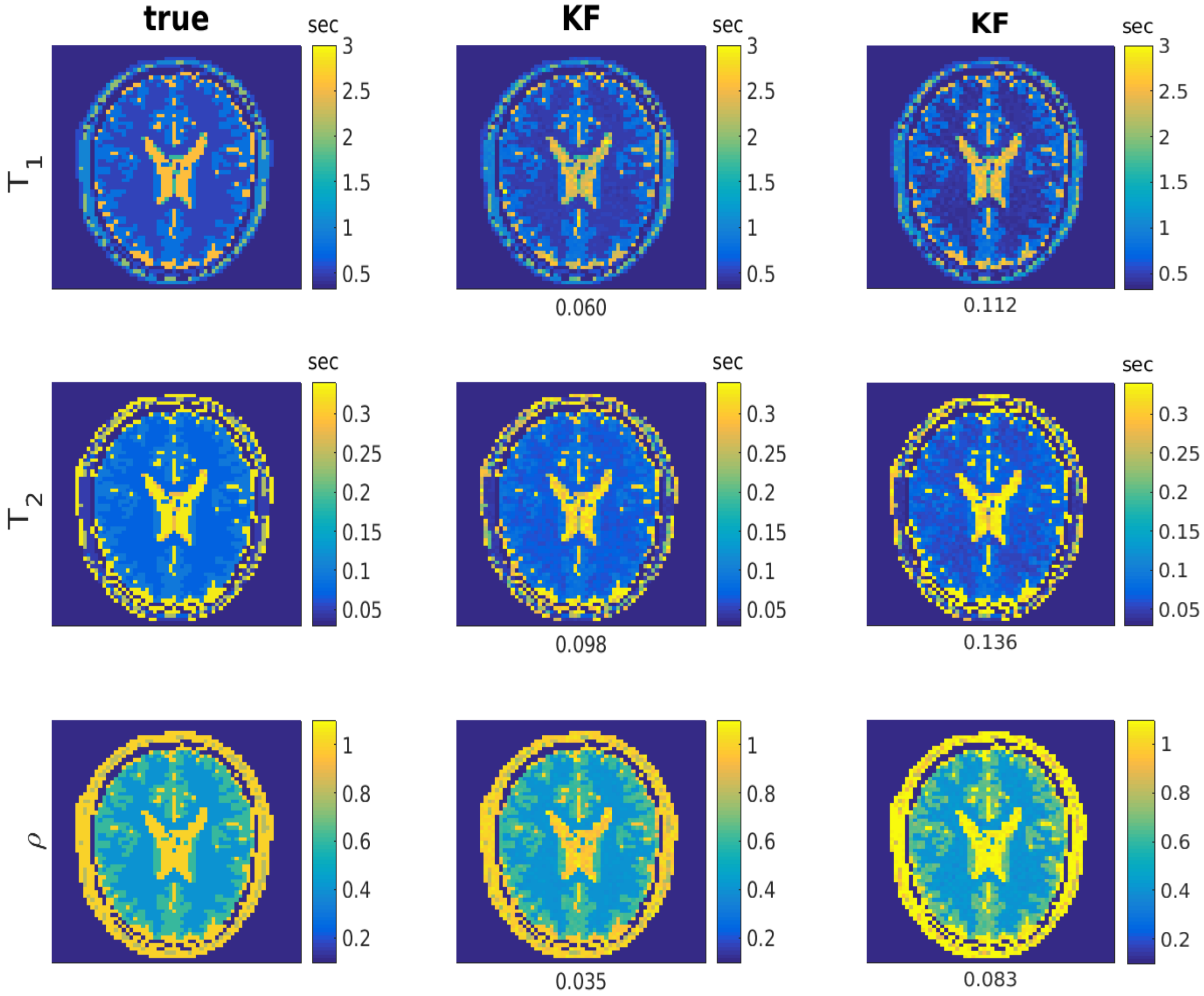
Gaussian Noise



4 x



Gaussian Noise 4 x



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