Robust and Scalable Iterative Solvers for Immersed Finite Element Methods

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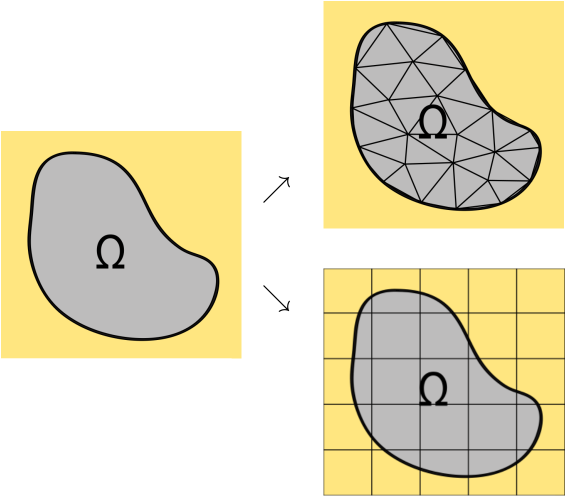
University of Colorado Boulder

Delft, May 30th, 2018

Outline

Concept of immersed methods (1): meshing

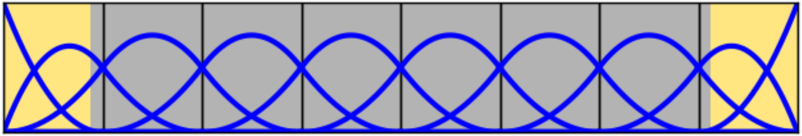


Conforming FEM

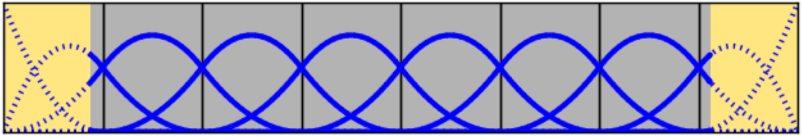
Immersed FEM

Concept of immersed methods (2): solution space

- **Step 1:** Basis functions defined on embedding domain:



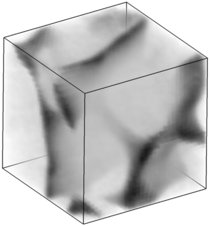
- **Step 2:** Basis functions trimmed to physical domain:

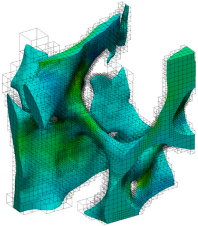


- **Step 3:** Approximation space spanned by trimmed basis functions

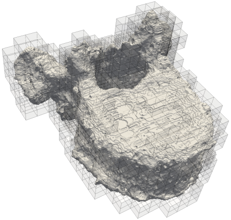
Application (1): elasticity on μCT-scanned porous bone structures

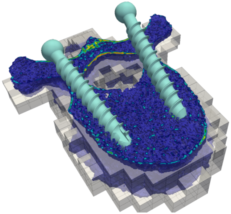
- Immersogeometric analysis on trabecular bone *[Verhoosel et. al. 2015]*:





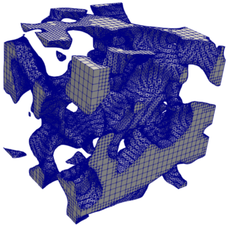
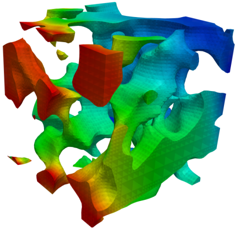
- Finite Cell Method on vertebra with implants *[Elhaddad et. al. 2017]*:



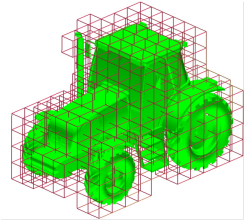
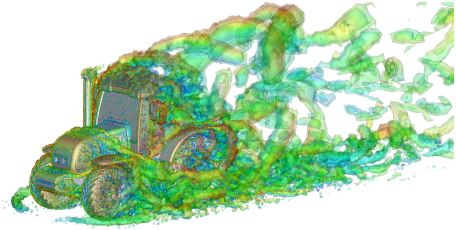


Application (2): flow on complex geometries

- Creeping flow through sinthered glass beads *[Hoang et. al. 2018]* (preprint):

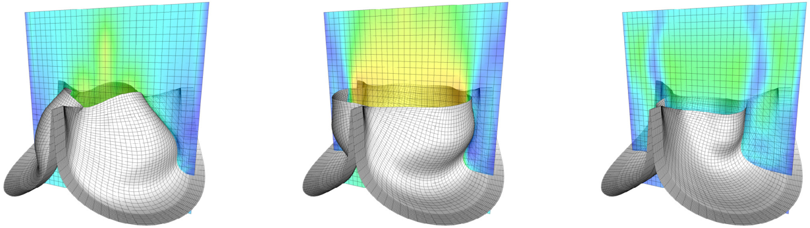



- Airflow around CAD model of a tractor *[Hsu et. al. 2016]*:

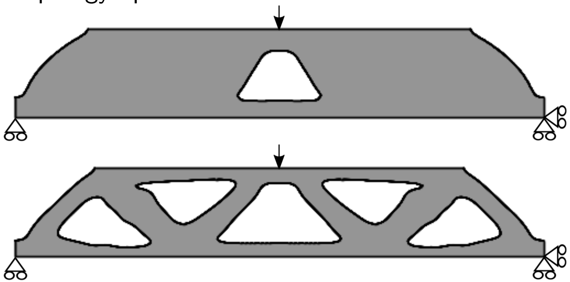



Application (3): moving domains with topology changes

- Simulation of bioprosthetic heart valves *[Kamensky et. al. 2017]*:

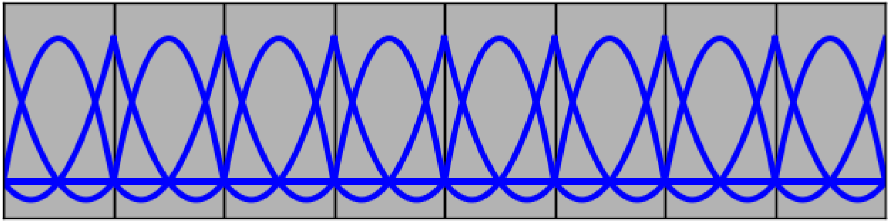


- Level set based topology optimization:



Outline

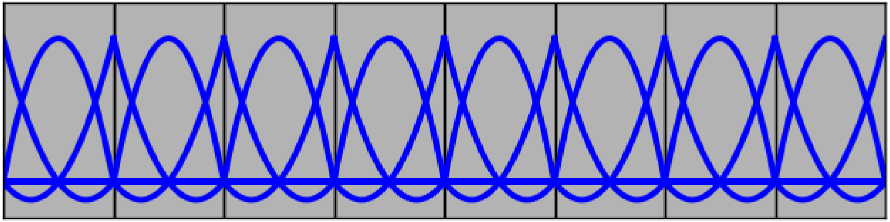
Conditioning analysis



function	v^h	$\Leftrightarrow$	$\mathbf{v}$	coefficient vector
weak form	$a(u^h, v^h) = b(v^h)$	$\Leftrightarrow$	$\mathbf{Au} = \mathbf{b}$	linear system

condition number: $\kappa(\mathbf{A}) = \|\mathbf{A}\|\|\mathbf{A}^{-1}\|$

Conditioning analysis



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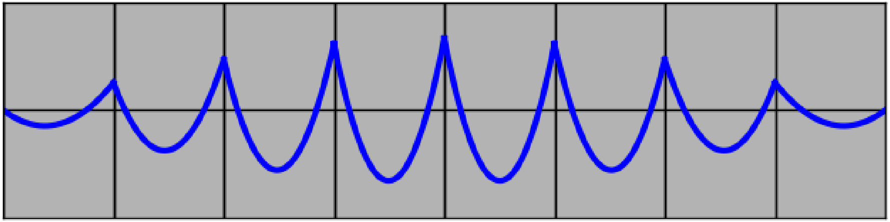

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$$\|\mathbf{A}\| = \max_{\mathbf{v}} \frac{\|\mathbf{A}\mathbf{v}\|}{\|\mathbf{v}\|} = \max_{\mathbf{v}} \frac{\mathbf{v}^T\mathbf{A}\mathbf{v}}{\mathbf{v}^T\mathbf{v}} = \max_{\mathbf{v}} \frac{a(v^h, v^h)}{\mathbf{v}^T\mathbf{v}}$$

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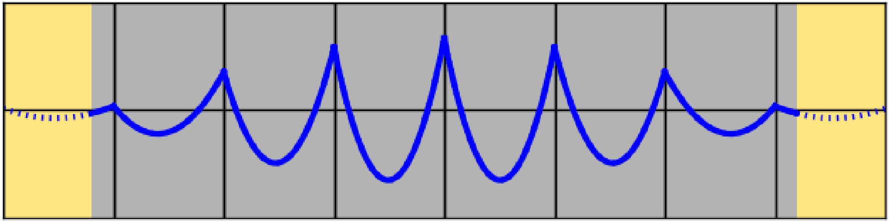

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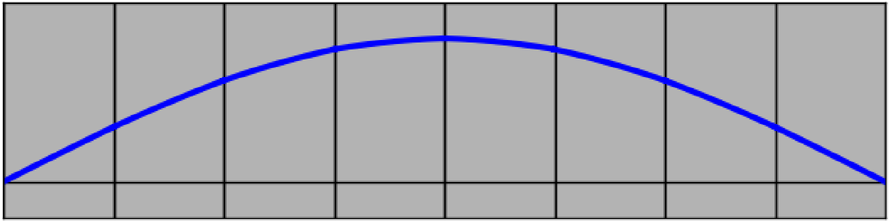

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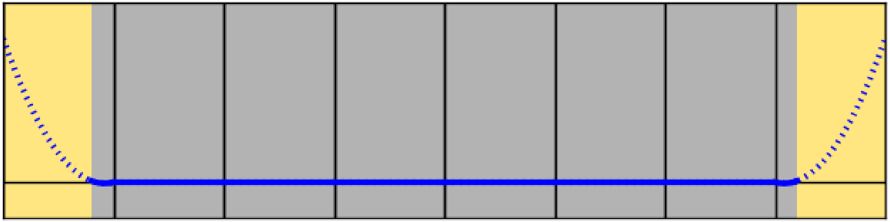

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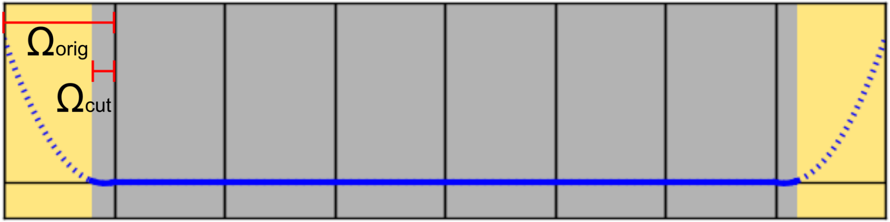

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$$\eta = \min_e \frac{|\Omega^e_{\text{cut}}|}{|\Omega^e_{\text{orig}}|} \qquad \kappa \propto \eta^{-(2p+1-2/d)}$$

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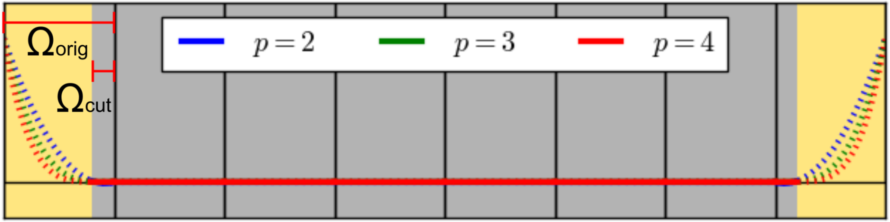

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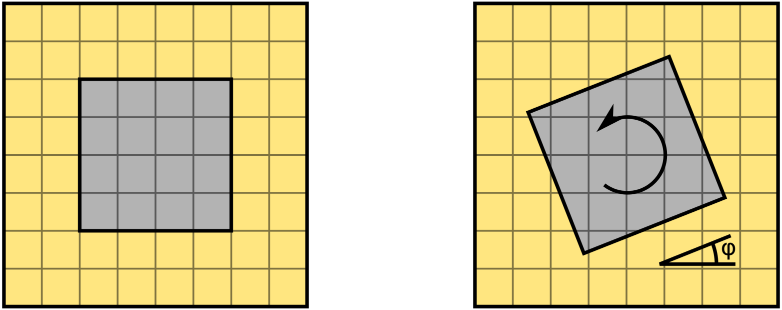
Example (1): setup

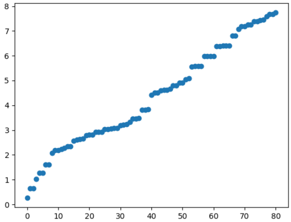


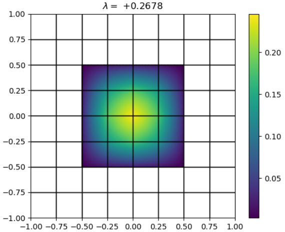
Problem setup:

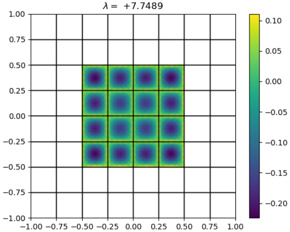
- Domain rotated over grid with $2^5 \times 2^5$ elements with a 2^{nd} order Lagrange basis
- *Different* discretizations of the *same* problem with the *same* mesh size
- Condition number and convergence for all separate rotations

Example (2): eigenmodes

eigenvalues





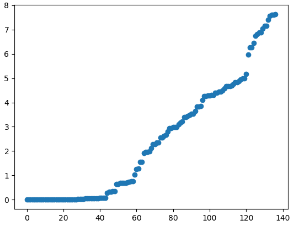


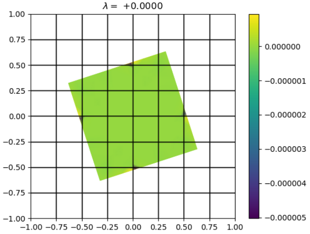
smallest eigenmode

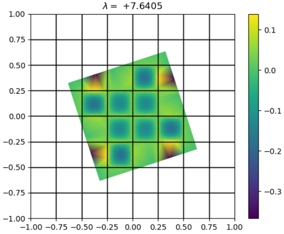
largest eigenmode

Example (2): eigenmodes

eigenvalues



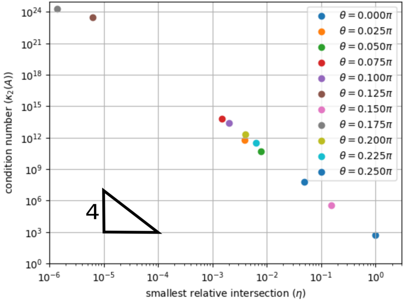


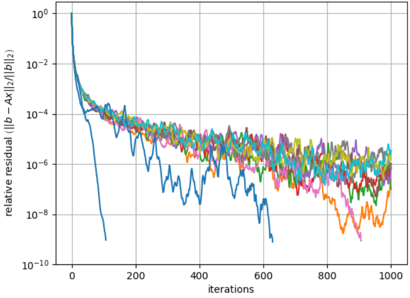


smallest eigenmode

largest eigenmode

Example (3): condition numbers and convergence

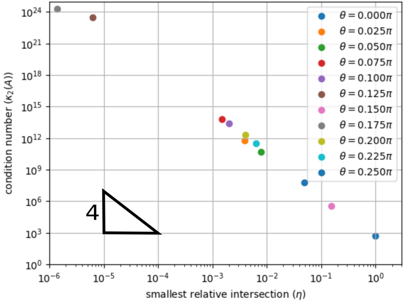




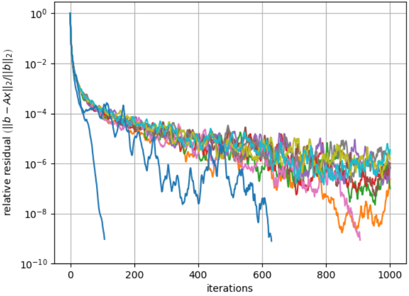
condition numbers

residual convergence of CG

Example (3): condition numbers and convergence



condition numbers



residual convergence of CG

Prenter, Verhoosel, Zwieten & Brummelen: Condition number analysis and preconditioning of the finite cell method, CMAME 2017

Jacobi preconditioning (1): results

unpreconditioned

Jacobi preconditioned

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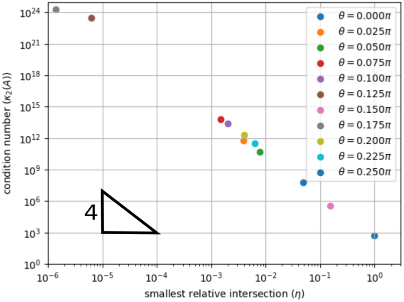
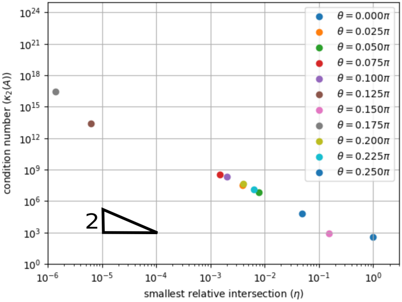
unpreconditioned

Jacobi preconditioned

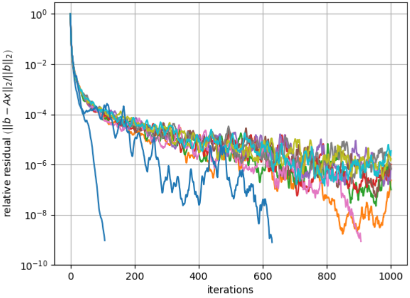




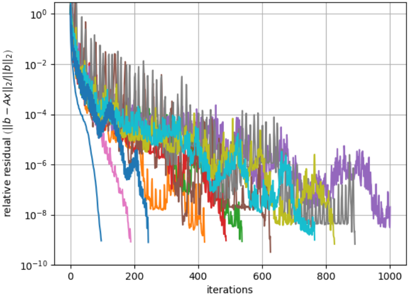
condition numbers

Jacobi preconditioning (1): results

unpreconditioned



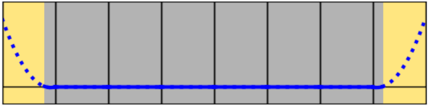
Jacobi preconditioned



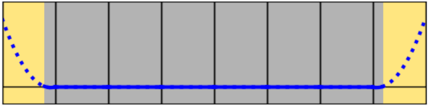
residual convergence of CG

Jacobi preconditioning (2): analysis

smallest mode
unpreconditioned



smallest mode
Jacobi preconditioned



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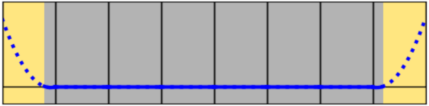

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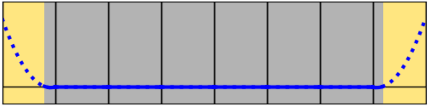

(almost) the same eigenfunction!

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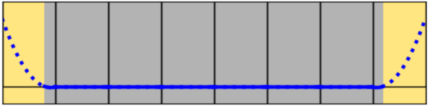

smallest mode
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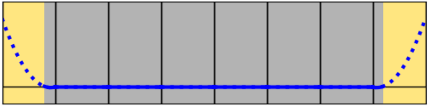

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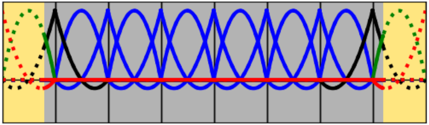
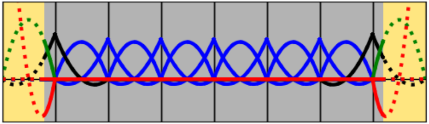
original basis

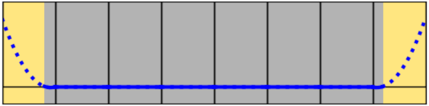


scaled basis

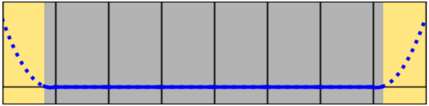


Jacobi preconditioning (2): analysis

smallest mode unpreconditioned



smallest mode Jacobi preconditioned



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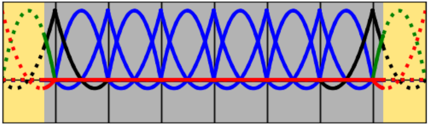
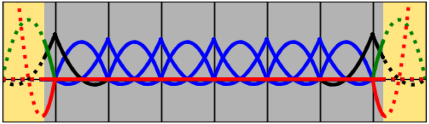


basis functions small and almost linearly dependent

scaled basis



basis functions scaled, but still almost linearly dependent!

Outline

Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}^{-1} \mathbf{P}_i$$

Additive-Schwarz preconditioning

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$$\mathbf{A} = \begin{bmatrix}
\bullet & \bullet & \bullet & & & & & & & \\
\bullet & \bullet & \bullet & \bullet & & & & & & \\
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& & & & & & & \bullet & \bullet & \bullet
\end{bmatrix} \Rightarrow \mathbf{M}^{-1} = \begin{bmatrix} \quad \end{bmatrix}$$

Additive-Schwarz preconditioning

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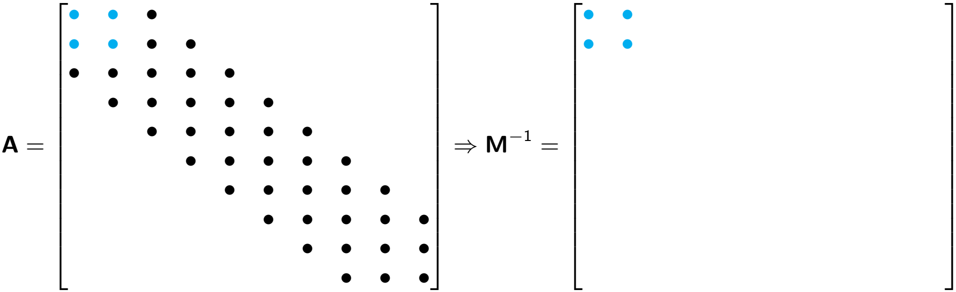

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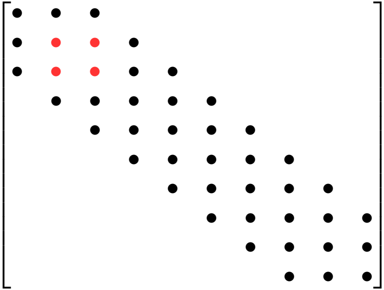
$\mathbf{A} =$



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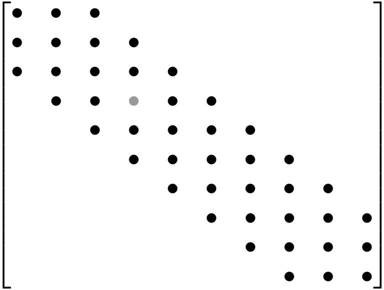

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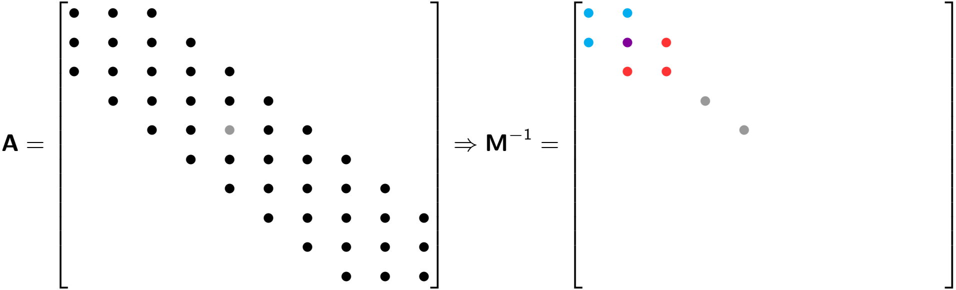

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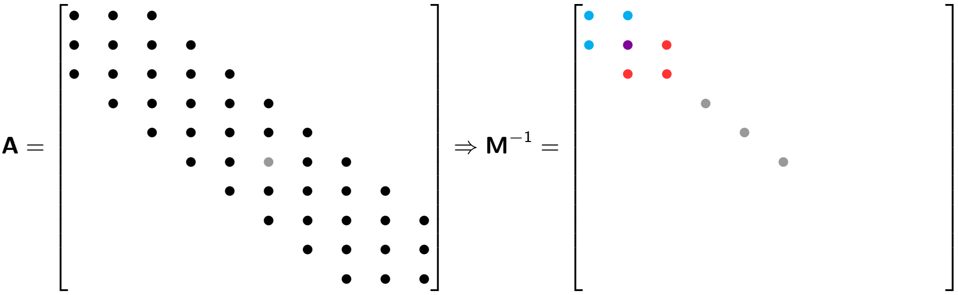
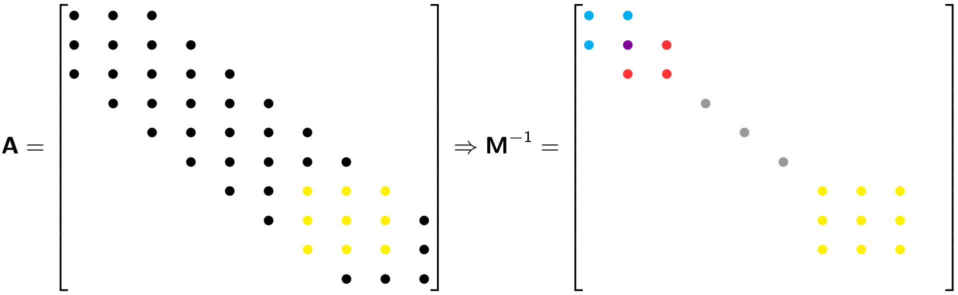
$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}{}^{-1} \mathbf{P}_i$$



Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}{}^{-1} \mathbf{P}_i$$



Additive-Schwarz preconditioning

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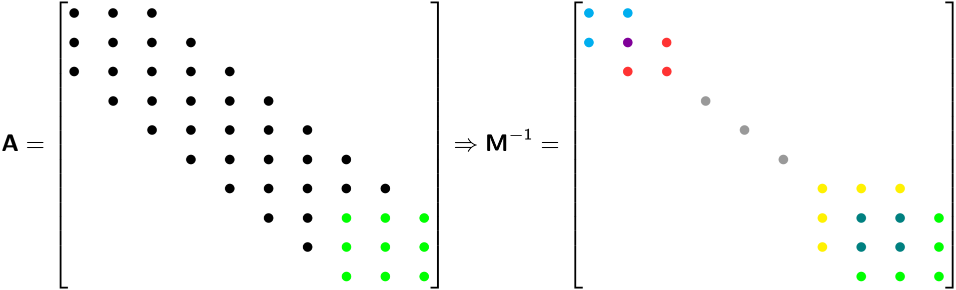

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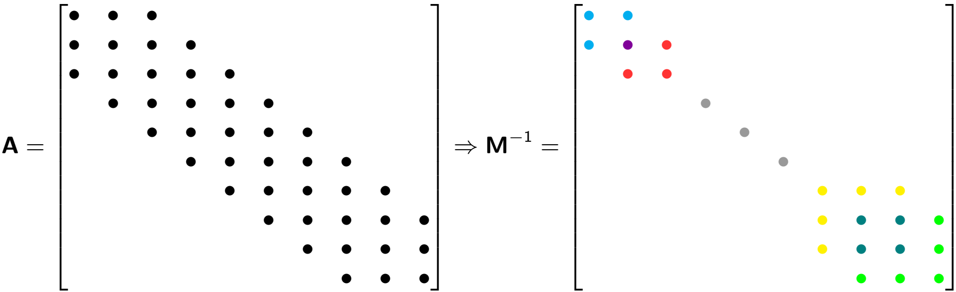

Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}^{-1} \mathbf{P}_i$$

$$\mathbf{v} = \sum_i \mathbf{P}_i^T \mathbf{v}_i$$

Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}{}^{-1} \mathbf{P}_i$$

$$\mathbf{v} = \sum_i \mathbf{P}_i^T \mathbf{v}_i$$

$$\mathbf{v}_1 = \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} \quad \Rightarrow \quad \mathbf{P}_1^T \mathbf{v}_1 = \begin{bmatrix} \bullet \\ \bullet \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} \quad \Rightarrow \quad \sum_{i=1}^{i=0} \mathbf{P}_i^T \mathbf{v}_i = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}{}^{-1} \mathbf{P}_i$$

$$\mathbf{v} = \sum_i \mathbf{P}_i^T \mathbf{v}_i$$

$$\mathbf{v}_1 = \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} \quad \Rightarrow \quad \mathbf{P}_1^T \mathbf{v}_1 = \begin{bmatrix} \bullet \\ \bullet \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} \quad \Rightarrow \quad \sum_{i=1}^{i=1} \mathbf{P}_i^T \mathbf{v}_i = \begin{bmatrix} \bullet \\ \bullet \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}^{-1} \mathbf{P}_i$$

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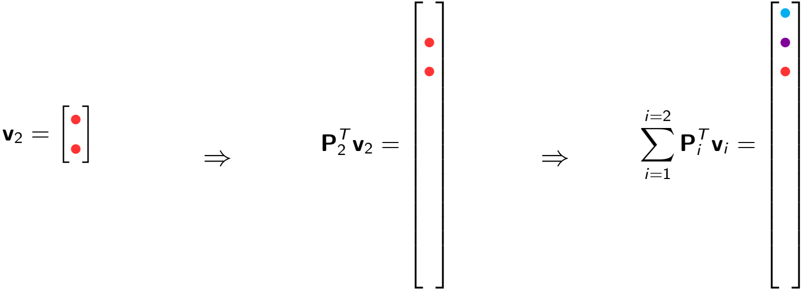

Additive-Schwarz preconditioning

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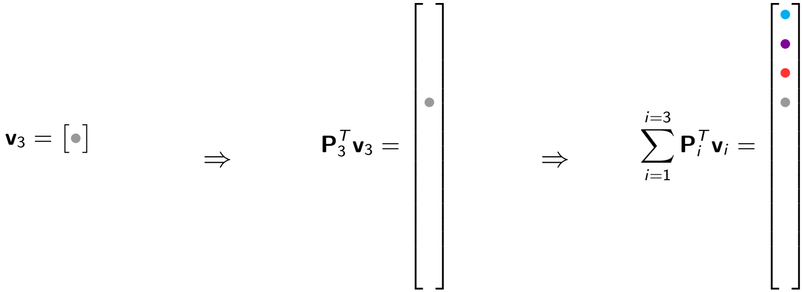

Additive-Schwarz preconditioning

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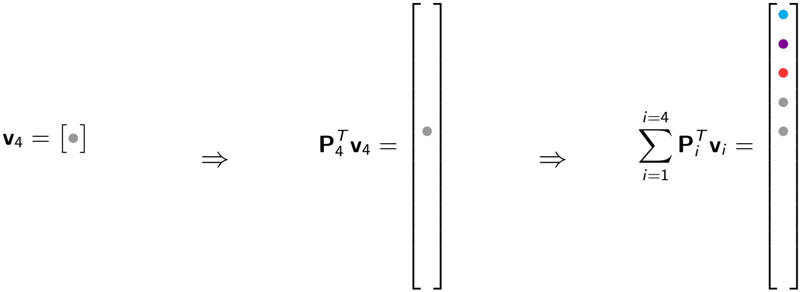

Additive-Schwarz preconditioning

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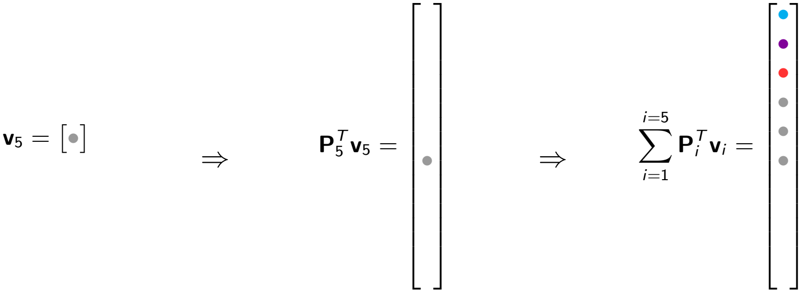

Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}^{-1} \mathbf{P}_i$$

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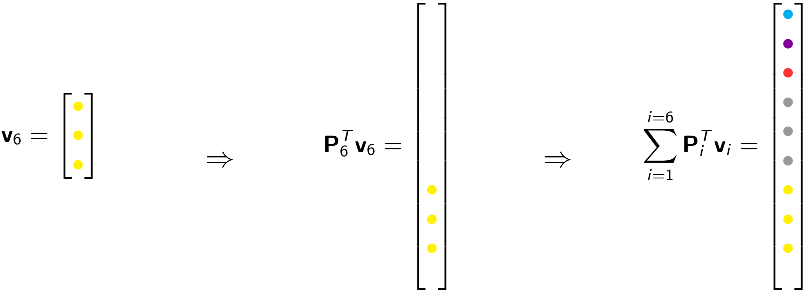

Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}{}^{-1} \mathbf{P}_i$$

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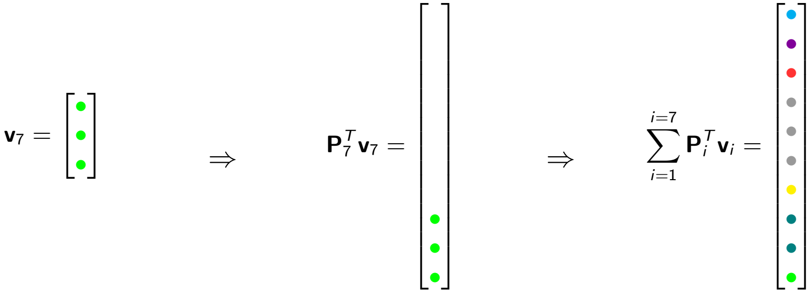


Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}^{-1} \mathbf{P}_i$$

$$\mathbf{v} = \sum_i \mathbf{P}_i^T \mathbf{v}_i$$

Additive-Schwarz lemma:

$$\mathbf{v}^T \mathbf{M} \mathbf{v} = \min_{\sum_i \mathbf{P}_i^T \mathbf{v}_i = \mathbf{v}} \sum_i \mathbf{v}_i^T \mathbf{A}_i \mathbf{v}$$

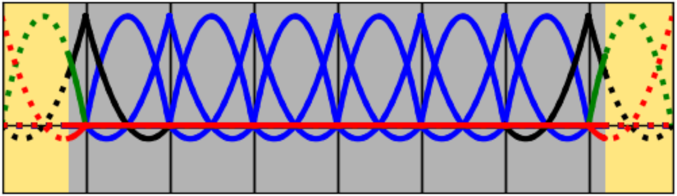
Additive-Schwarz preconditioning

$$\mathbf{M}^{-1} = \sum_i \mathbf{P}_i^T \underbrace{\left(\mathbf{P}_i \mathbf{A} \mathbf{P}_i^T\right)}_{\mathbf{A}_i}^{-1} \mathbf{P}_i$$

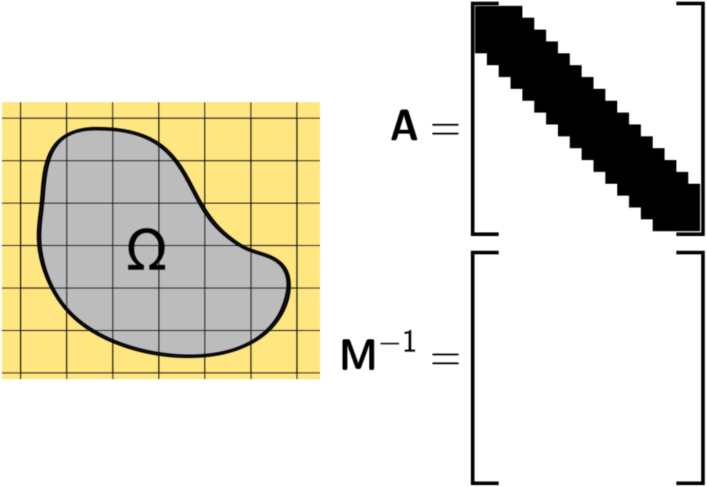
$$\mathbf{v} = \sum_i \mathbf{P}_i^T \mathbf{v}_i$$

Additive-Schwarz lemma:

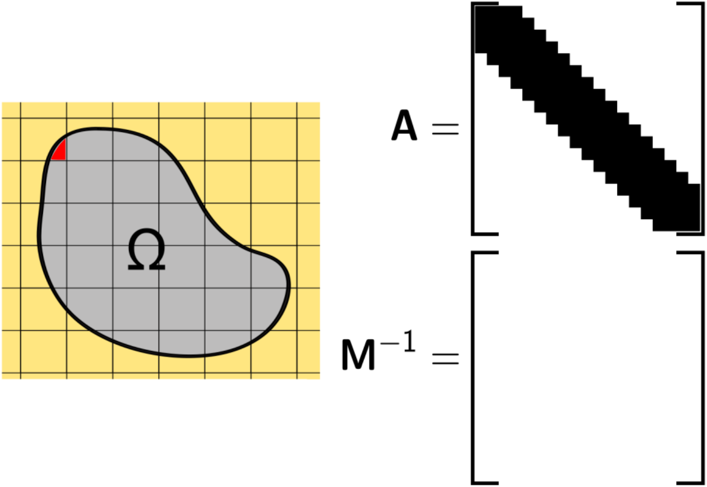
$$\mathbf{v}^T \mathbf{M} \mathbf{v} = \min_{\sum_i \mathbf{P}_i^T \mathbf{v}_i = \mathbf{v}} \sum_i \mathbf{v}_i^T \mathbf{A}_i \mathbf{v}$$



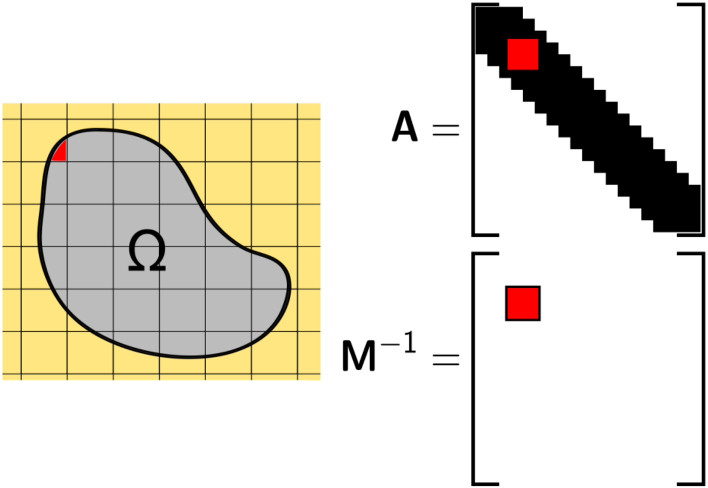
Setting blocks



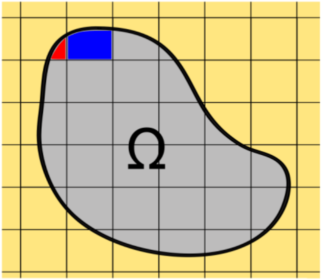
Setting blocks

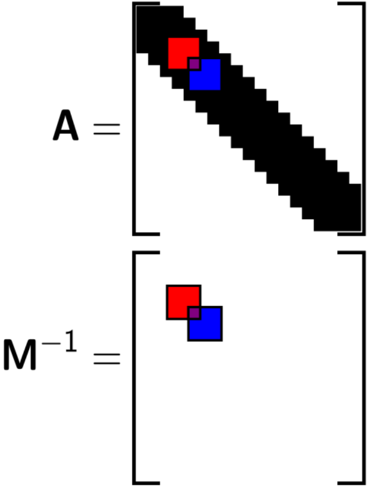


Setting blocks

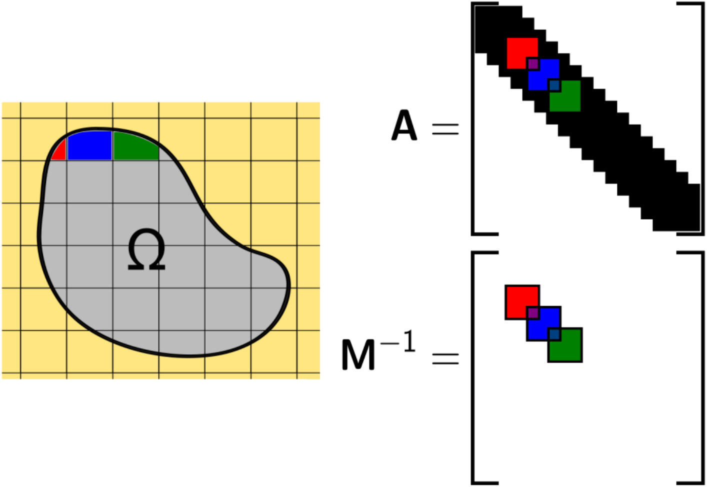


Setting blocks

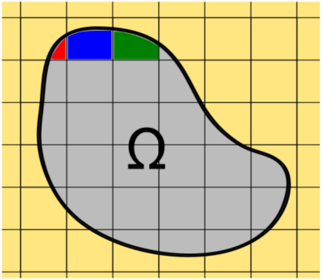


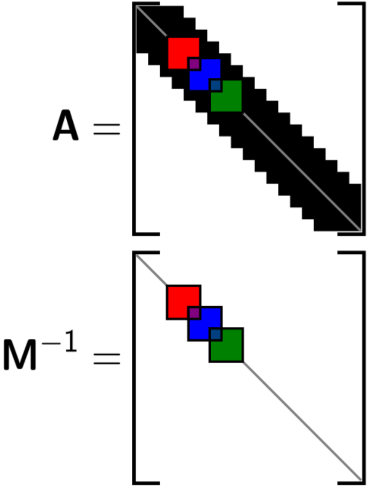


Setting blocks

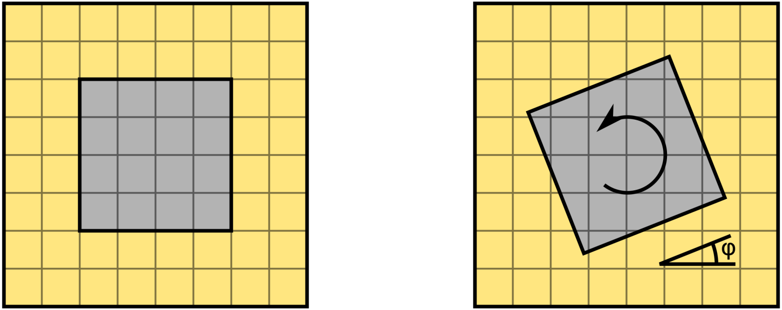


Setting blocks





Example revisited (1): setup

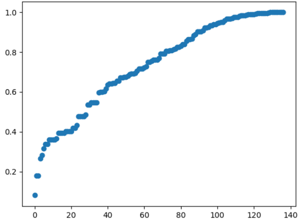


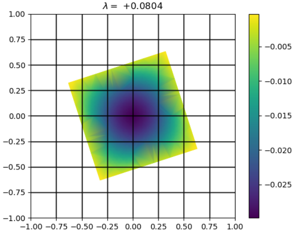
Problem setup:

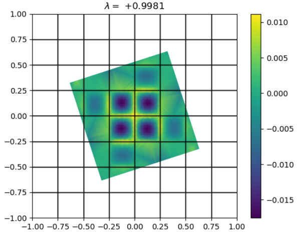
- Domain rotated over grid with $\mathbf{2^n} \times \mathbf{2^n}$ elements with a 2^{nd} order Lagrange basis
- *Different* discretizations of the *same* problem with the *same* mesh size
- Condition number and convergence for **the preconditioned system**

Example revisited (2): eigenmodes

eigenvalues





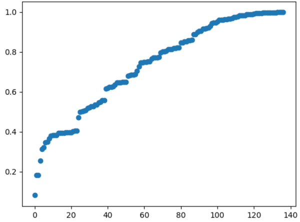


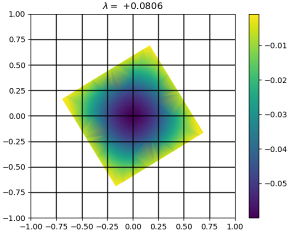
smallest eigenmode

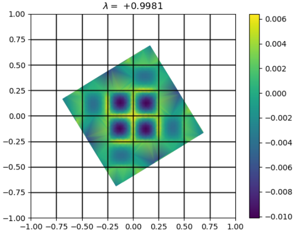
largest eigenmode

Example revisited (2): eigenmodes

eigenvalues





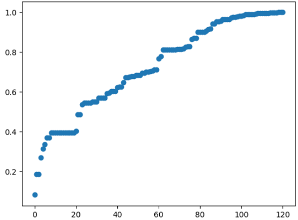


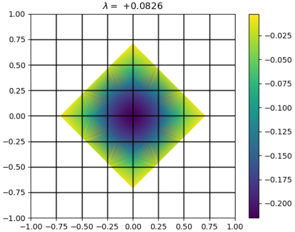
smallest eigenmode

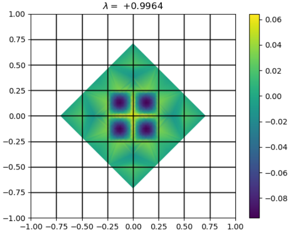
largest eigenmode

Example revisited (2): eigenmodes

eigenvalues



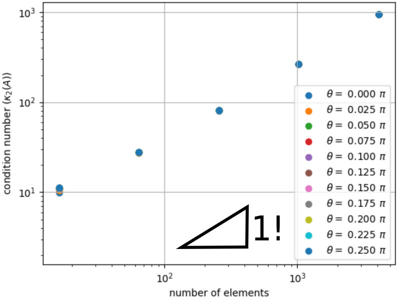




smallest eigenmode

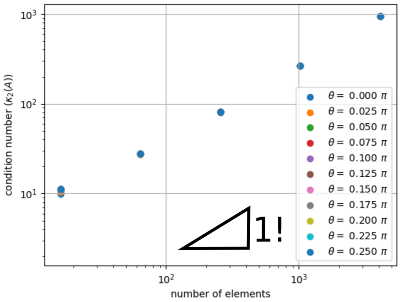
largest eigenmode

Example revisited (3): condition numbers and convergence

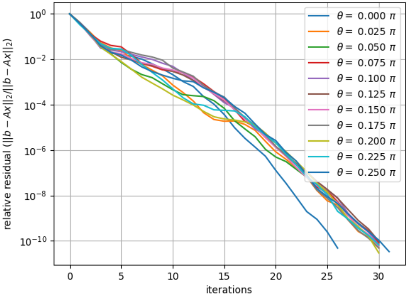


condition numbers

Example revisited (3): condition numbers and convergence

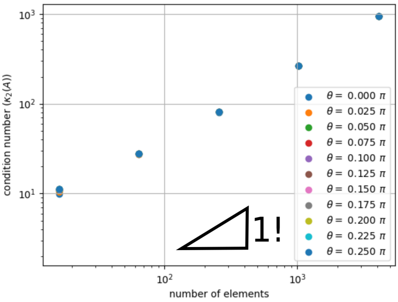


condition numbers

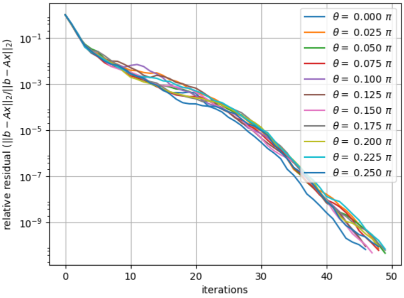


residual convergence of CG
$2^3 \times 2^3$ elements

Example revisited (3): condition numbers and convergence

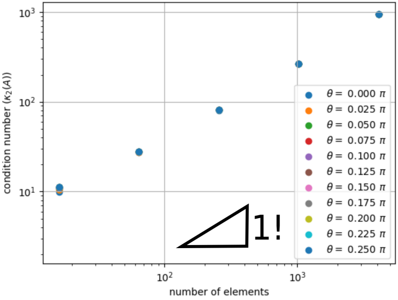


condition numbers

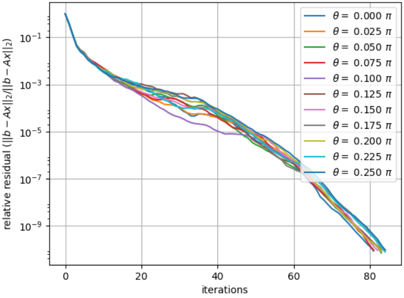


residual convergence of CG
$2^4 \times 2^4$ elements

Example revisited (3): condition numbers and convergence

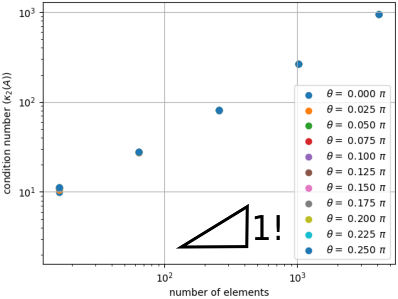


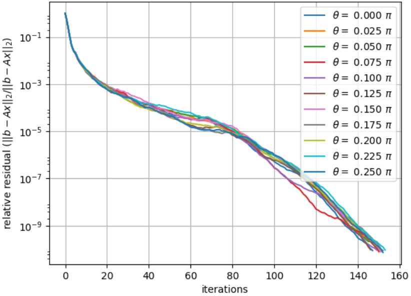
condition numbers



residual convergence of CG
$2^5 \times 2^5$ elements

Example revisited (3): condition numbers and convergence





condition numbers

residual convergence of CG
$2^6 \times 2^6$ elements

Outline

Multigrid V-cycle

Thank you Prof. Oosterlee!

Multigrid V-cycle

Thank you Prof. Oosterlee!

Multigrid V-cycle

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Multigrid V-cycle

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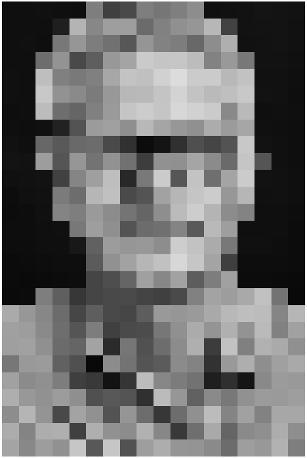
Multigrid V-cycle

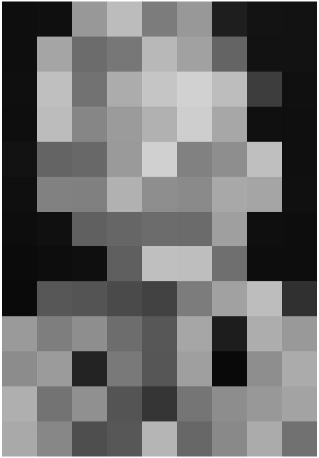
Thank you Prof. Oosterlee!

Multigrid V-cycle



Thank you Prof. Oosterlee!

Multigrid V-cycle



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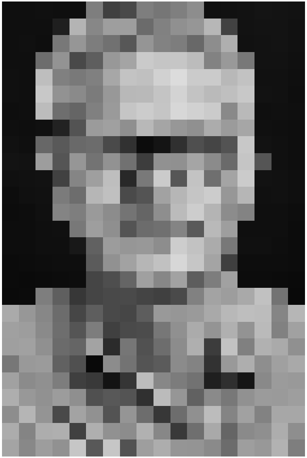

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Multigrid V-cycle

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Multigrid V-cycle

Thank you Prof. Oosterlee!

Thank you Prof. Oosterlee!

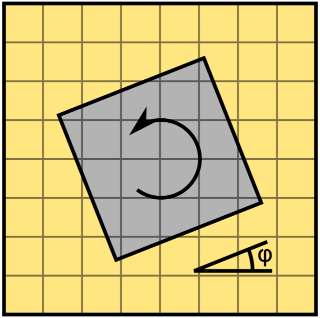
Multigrid V-cycle

Thank you Prof. Oosterlee!

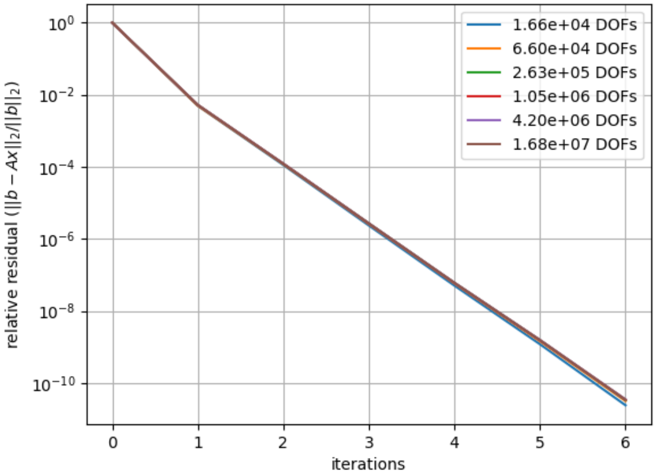
Example (1): problem setup

Problem setup:

- Domain at $\phi = 22.5^\circ$ with background grid
- $2^n \times 2^n$ elements for $n \in \{6, 7, 8, 9, 10, 11\}$
- $\{3, 4, 5, 6, 7, 8\}$ levels
- 2^{nd} order Lagrange basis
- $\approx \frac{1}{4}$ of basis functions active
- Preconditioned CG solver
- V-cycle smoothed with Multiplicative-Schwarz/ Gauss-Seidel scheme

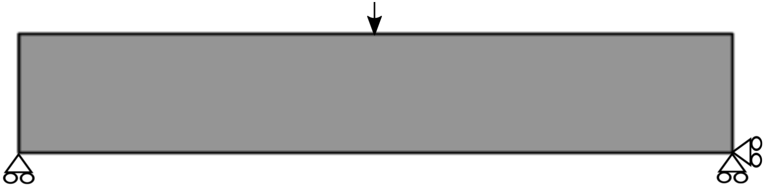


Example (2): convergence results



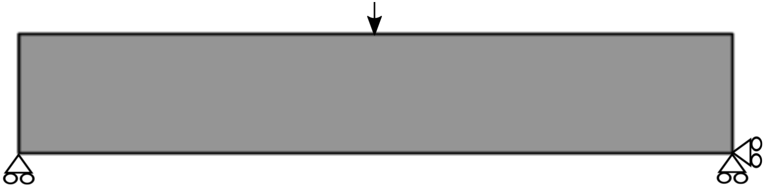
Outline

Level set based optimization of beam

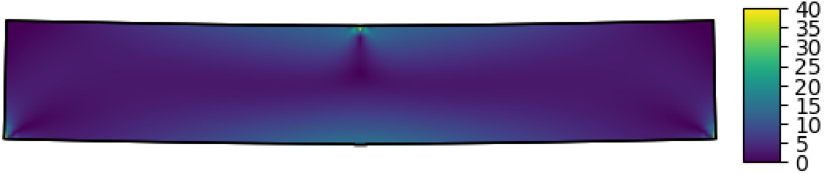


initial geometry

Level set based optimization of beam

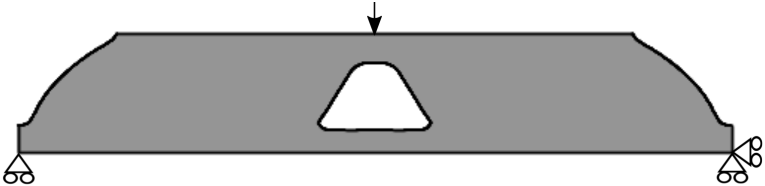


initial geometry

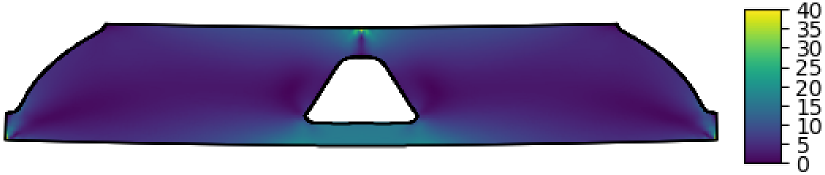


displacement and stress

Level set based optimization of beam

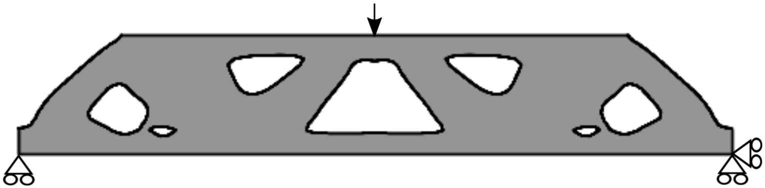


geometry after 30 iterations

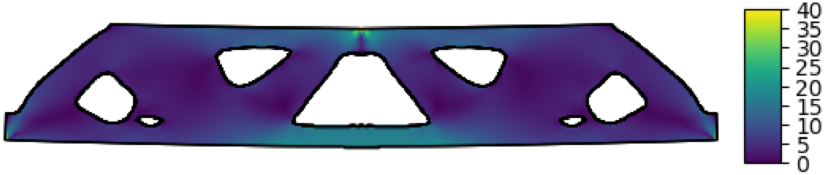


displacement and stress

Level set based optimization of beam

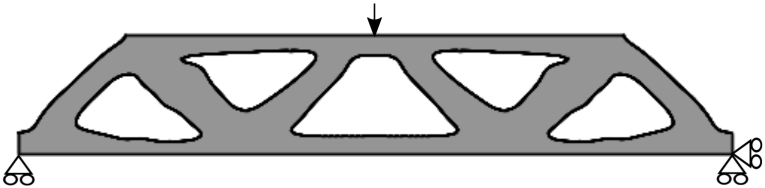


geometry after 40 iterations

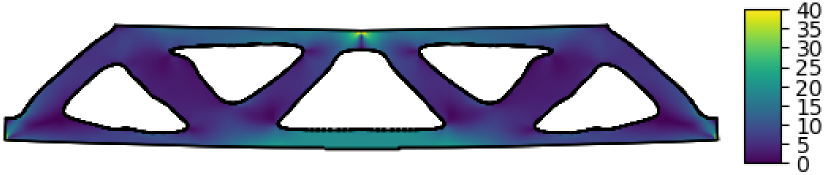


displacement and stress

Level set based optimization of beam

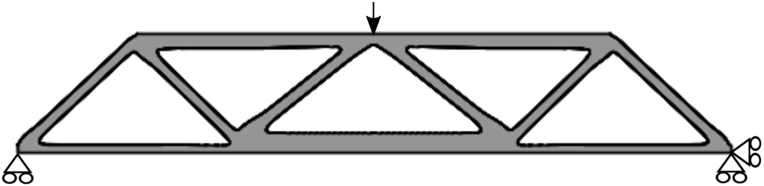


geometry after 50 iterations

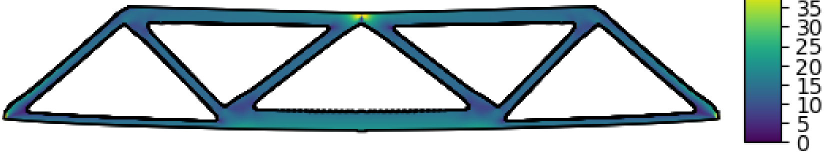


displacement and stress

Level set based optimization of beam

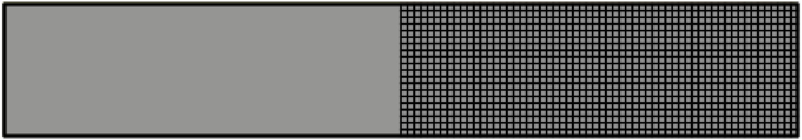


geometry after 150 iterations



displacement and stress

Immersed discretization

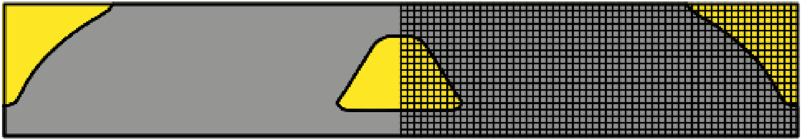


initial geometry and mesh

Mesh details:

- Half of the problem solved using symmetry in the vertical plane
- $\{60 \times 20, 120 \times 40, 240 \times 80\}$ elements at coarsest levels
- $\{2, 3, 4\}$ levels of coarsening
- 1 level of refinement after 50 iterations
- 2 levels of refinement after 125 iterations
- Truncated hierarchical quadratic B-splines

Immersed discretization

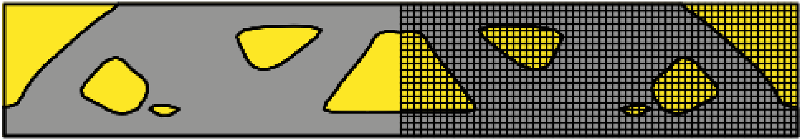


geometry and mesh after 30 iterations

Mesh details:

- Half of the problem solved using symmetry in the vertical plane
- $\{60 \times 20, 120 \times 40, 240 \times 80\}$ elements at coarsest levels
- $\{2, 3, 4\}$ levels of coarsening
- 1 level of refinement after 50 iterations
- 2 levels of refinement after 125 iterations
- Truncated hierarchical quadratic B-splines

Immersed discretization

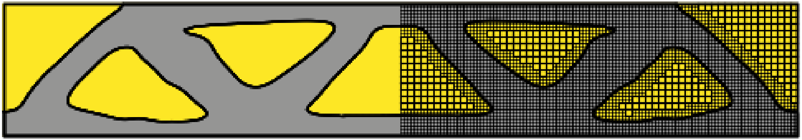


geometry and mesh after 40 iterations

Mesh details:

- Half of the problem solved using symmetry in the vertical plane
- $\{60 \times 20, 120 \times 40, 240 \times 80\}$ elements at coarsest levels
- $\{2, 3, 4\}$ levels of coarsening
- 1 level of refinement after 50 iterations
- 2 levels of refinement after 125 iterations
- Truncated hierarchical quadratic B-splines

Immersed discretization

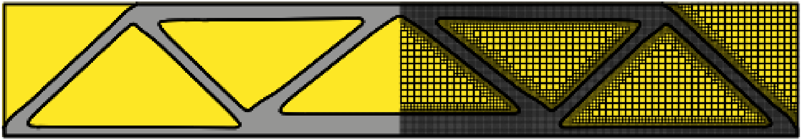


geometry and mesh after 50 iterations

Mesh details:

- Half of the problem solved using symmetry in the vertical plane
- $\{60 \times 20, 120 \times 40, 240 \times 80\}$ elements at coarsest levels
- $\{2, 3, 4\}$ levels of coarsening
- 1 level of refinement after 50 iterations
- 2 levels of refinement after 125 iterations
- Truncated hierarchical quadratic B-splines

Immersed discretization

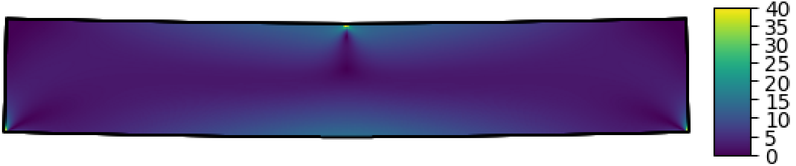


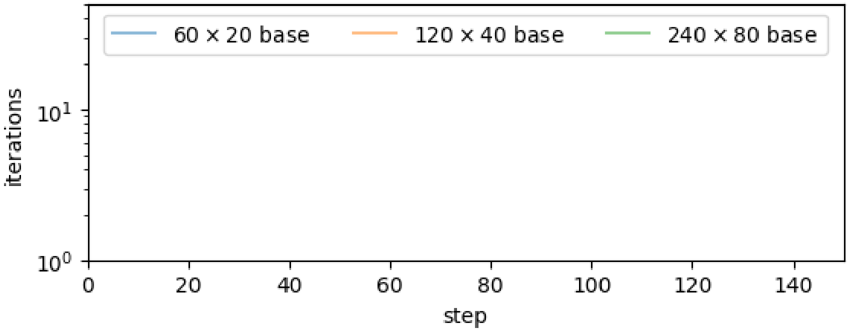
geometry and mesh after 150 iterations

Mesh details:

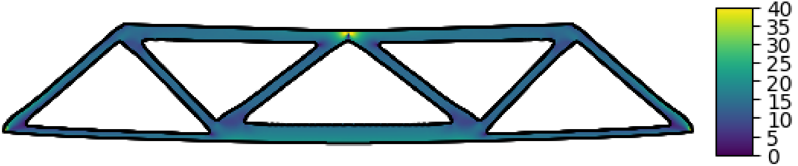
- Half of the problem solved using symmetry in the vertical plane
- $\{60 \times 20, 120 \times 40, 240 \times 80\}$ elements at coarsest levels
- $\{2, 3, 4\}$ levels of coarsening
- 1 level of refinement after 50 iterations
- 2 levels of refinement after 125 iterations
- Truncated hierarchical quadratic B-splines

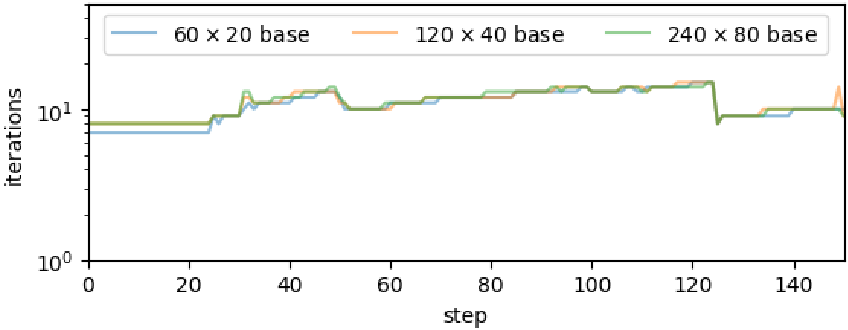
Iteratively solving the linear systems





Iteratively solving the linear systems





Outline

Summary and outlook

Summary

- Preconditioner for immersed finite elements that is robust to:
  - how elements are cut
  - the mesh size

Outlook

- Testing on three-dimensional testcases and scanned data
- Extending the procedure for fluid problems

Robust and Scalable Iterative Solvers for Immersed Finite Element Methods

Frits de Prenter, Clemens Verhoosel, Harald van Brummelen, John Evans, Joseph Benzaken & Christian Messe

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