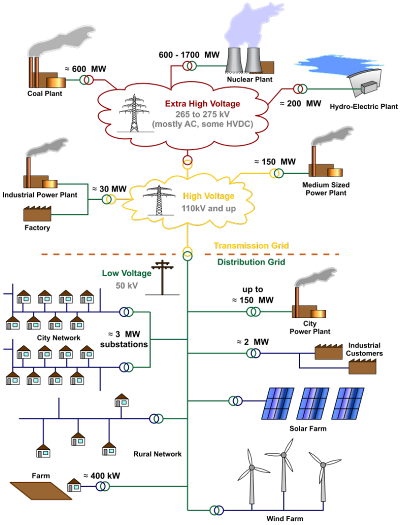
Solving the non-linear power flow equations using the physical properties of the power system

B. Sereeter[†]

[†]TJIP B.V., Informaticalaan 5, 2628ZD Delft, The Netherlands

October 15, 2020

Power grid



≈ 600 MW
Coal Plant
600 - 1700 MW
Nuclear Plant
Extra High Voltage
265 to 275 kV
(mostly AC, some HVDC)
≈ 200 MW
Hydro-Electric Plant
≈ 150 MW
Medium Sized
Power Plant
Industrial Power Plant
≈ 30 MW
Factory
High Voltage
110kV and up
Transmission Grid
Distribution Grid
Low Voltage
50 kV
up to
≈ 150 MW
City
Power Plant
City Network
≈ 3 MW
substations
≈ 2 MW
Industrial
Customers
Solar Farm
Rural Network
Farm
≈ 400 kW
Wind Farm

Power flow problem

The mathematical equations for the power flow problem are given by:

$$S_i = V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* \tag{1}$$

where N is a number of buses in the network, S_i injected complex power, V_i is the complex voltage, and Y_{ij} is an element of the admittance matrix.

Solution methods

- Gauss-Seidel (G-S)
- **Newton power flow (NR)**
- Fast Decoupled Load flow (FDLF)
- Backward-Forward Sweep (BFS) based-algorithms

Newton based power flow methods

$$S_i = V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* \quad (\Rightarrow) \quad F(x) = 0 \quad (\Rightarrow) - J(x)\Delta x = F(x)$$

$$\begin{bmatrix} \text{Power balance:} & F(x) = S_i^{sp} - V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* = 0 \\ \text{Current balance:} & F(x) = \left(\frac{S_i^{sp}}{V_i}\right)^* - \sum_{k=1}^{N} Y_{ik} V_k = 0 \end{bmatrix}$$

Power balance equations in **polar** coordinates (**PP**):

$$F(x) = \begin{bmatrix} P_i^{sp} - \sum_{k=1}^{N} |V_i||V_k|\big(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}\big) = 0 \\ Q_i^{sp} - \sum_{k=1}^{N} |V_i||V_k|\big(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}\big) = 0 \end{bmatrix}$$

Newton based power flow methods

$$S_i = V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* \quad (\Rightarrow) \quad F(\mathrm{x}) = 0 \quad (\Rightarrow) - J(\mathrm{x})\Delta \mathrm{x} = F(\mathrm{x})$$

$$\begin{bmatrix} \text{Power balance:} & F(\mathrm{x}) = S_i^{sp} - V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* = 0 \\ \text{Current balance:} & F(\mathrm{x}) = \left(\frac{S_i^{sp}}{V_i}\right)^* - \sum_{k=1}^{N} Y_{ik} V_k = 0 \end{bmatrix}$$

Power balance equations in **polar** coordinates (**PP**):

$$F(\mathrm{x}) = \begin{bmatrix} P_i^{sp} - \sum_{k=1}^{N} |V_i||V_k|\big(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}\big) = 0 \\ Q_i^{sp} - \sum_{k=1}^{N} |V_i||V_k|\big(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}\big) = 0 \end{bmatrix}$$

Newton based power flow methods

$$S_i = V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* \quad (\Rightarrow) \quad F(\mathrm{x}) = 0 \quad (\Rightarrow) - J(\mathrm{x})\Delta \mathrm{x} = F(\mathrm{x})$$

$$\begin{bmatrix} \text{Power balance:} & F(\mathrm{x}) = S_i^{sp} - V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* = 0 \\ \text{Current balance:} & F(\mathrm{x}) = \left(\frac{S_i^{sp}}{V_i}\right)^* - \sum_{k=1}^{N} Y_{ik} V_k = 0 \end{bmatrix}$$

Power balance equations in **polar** coordinates (**PP**):

$$F(\mathrm{x}) = \begin{bmatrix} P_i^{sp} - \sum_{k=1}^{N} |V_i||V_k| \left(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}\right) = 0 \\ Q_i^{sp} - \sum_{k=1}^{N} |V_i||V_k| \left(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}\right) = 0 \end{bmatrix}$$

Mathematical formulations

Power mismatch equations in **Cartesian** coordinates (**PC**):

$$\begin{bmatrix} P_i^{sp} - \sum_{k=1}^{N} \Big(V_i^r (G_{ik} V_k^r - B_{ik} V_k^m) + V_i^m (B_{ik} V_k^r + G_{ik} V_k^m) \Big) = 0 \\ Q_i^{sp} - \sum_{k=1}^{N} \Big(V_i^m (G_{ik} V_k^r - B_{ik} V_k^m) - V_i^r (B_{ik} V_k^r + G_{ik} V_k^m) \Big) = 0 \end{bmatrix}$$

Current mismatch equations in **polar** coordinates (**CP**):

$$\begin{bmatrix} \frac{P_i^{sp} \cos \delta_i + Q_i^{sp} \sin \delta_i}{|V_i|} - \sum_{k=1}^{N} |V_k| \big(G_{ik} \cos \delta_k - B_{ik} \sin \delta_k \big) = 0 \\ \frac{P_i^{sp} \sin \delta_i - Q_i^{sp} \cos \delta_i}{|V_i|} - \sum_{k=1}^{N} |V_k| \big(G_{ik} \sin \delta_k + B_{ik} \cos \delta_k \big) = 0 \end{bmatrix}$$

Current mismatch equations in **Cartesian** coordinates (**CC**):

$$\begin{bmatrix} \frac{P_i^{sp} V_i^r + Q_i^{sp} V_i^m}{(V_i^r)^2 + (V_i^m)^2} - \sum_{k=1}^{N} (G_{ik} V_k^r - B_{ik} V_k^m) = 0 \\ \frac{P_i^{sp} V_i^m - Q_i^{sp} V_i^r}{(V_i^r)^2 + (V_i^m)^2} - \sum_{k=1}^{N} (G_{ik} V_k^m + B_{ik} V_k^r) = 0 \end{bmatrix}$$

Newton power flow methods (NR)

Mismatch formulation	Coordinates	
	Polar (-pol)	Cartesian (-car)
Power (NR-p)	**[1]**†	[2]
Current (NR-c)	[3]	[3, 4]

Table: Known versions of the Newton power flow method

Numerical results

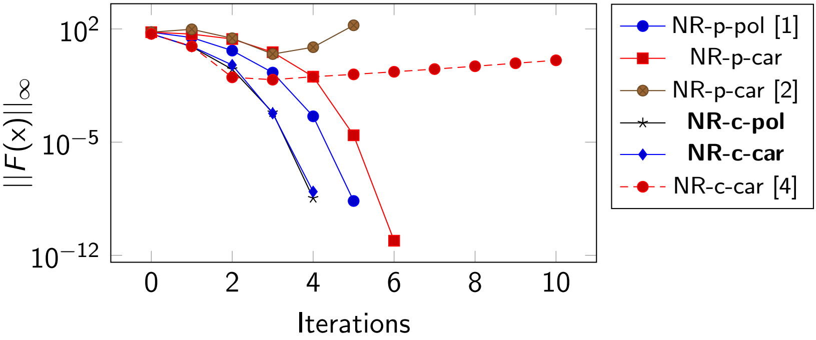


Figure: Large transmission network: TCase13659

Convergence for different R/X ratios

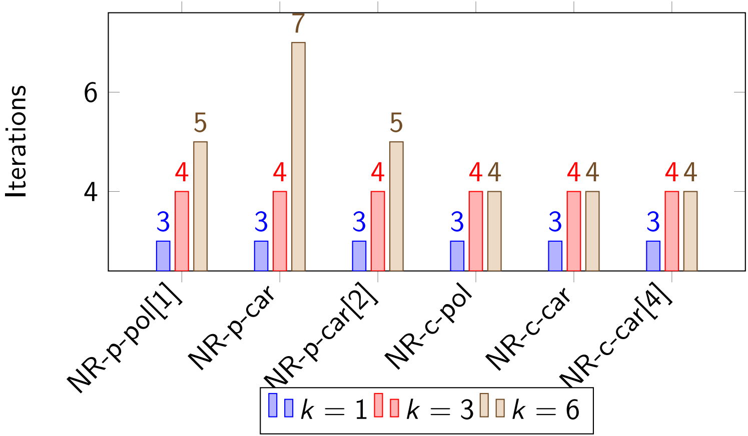


Figure: UDCase37: Different R/X ratios as $Z = k * R + \imath X$

Physical formulation of the OPF problem

The objective is to minimize the total cost for P^g:

$$\min_{\mathrm{x}} \; f(\mathrm{x}, \mathrm{u}) = \sum_{i=1}^{N_g} \left(C_i^0 + C_i^1 P_i^g + C_i^2 (P_i^g)^2 \right) \tag{2}$$

subject to equality constraints $g(\mathrm{x}, \mathrm{u})$ as power flow equations:

$$g_i(\mathrm{x}, \mathrm{u}) = S_i - V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* = 0 \tag{3}$$

and inequality constraints $h(\mathrm{x}, \mathrm{u})$ as squared branch flow limits:

$$h_{ij}(\mathrm{x}, \mathrm{u}) = \begin{bmatrix} |S_{ij}^f(\mathrm{x}, \mathrm{u})|^2 \\ |S_{ij}^t(\mathrm{x}, \mathrm{u})|^2 \end{bmatrix} \leq \begin{bmatrix} (S_{ij}^{\max})^2 \\ (S_{ij}^{\max})^2 \end{bmatrix} \tag{4}$$

Solution methods

1. Deterministic
   - Gradient Methods
   - Hessian Methods
   - Sequential Linear Programming (SLP)
   - Sequential Quadratic Programming (SQP)
   - **Interior Point Methods (IPM)**
2. Non-Deterministic
   - Artificial Neural Networks (ANN)
   - Fuzzy Logic
   - Evolutionary Programming
   - Ant Colony Optimization (ACO)
   - Particle Swarm Optimization (PSO)

Interior Point Method (IPM)

The IPM algorithm assembles the object function, equality, and inequality constraints into the linearized Karush-Kuhn-Tucker (KKT) conditions:

$$\begin{bmatrix} \mathcal{L}_{xx}^{\gamma} & 0 & g_x^T & h_x^T \\ 0 & [\mu] & 0 & [z] \\ g_x & 0 & 0 & 0 \\ h_x & I & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \\ \Delta \lambda \\ \Delta \mu \end{bmatrix} = - \begin{bmatrix} f_x^T + g_x^T \lambda + h_x^T \mu \\ [\mu]z - \gamma e \\ g(x) \\ h(x) + z \end{bmatrix} \tag{5}$$

where

$$\mathcal{L}_{xx}^{\gamma} = f_{xx} + g_{xx}(\lambda) + h_{xx}(\mu).$$

IPM: Number of iterations

Variant	Test cases						
	c89	c118	c300	c588	c2383	c2736	c3120
PP	NC	20	**18**	41	**33**	28	108
PC	**26**	21	31	37	37	34	54
CP	30	**19**	**18**	**35**	**33**	**27**	**45**
CC	NC	20	22	37	35	35	50

Table: Number of iterations of IPM on various test cases

LPF problem vs NPF problem

Recall the mathematical equations for the nonlinear power flow problem:

$$S_i = V_i I_i^* = V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* \tag{6}$$

where

$$I = YV. \tag{7}$$

LPF model vs NPF model

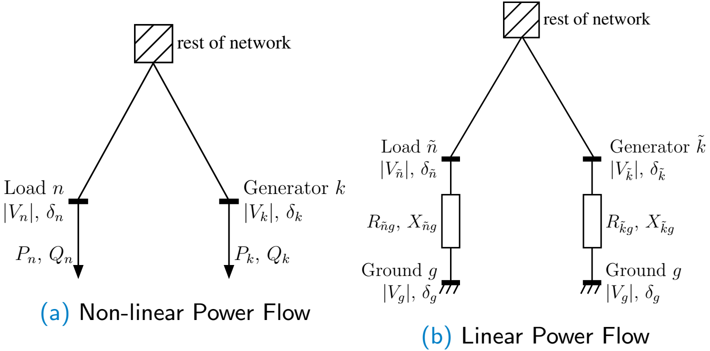


Figure: Network representation of a load bus n and a generator bus k for standard NPF (a) and for LPF (b).

LPF: Impedance for additional lines

Resistance R_{ig} and reactance X_{ig} for the additional lines are computed by:

$$\begin{aligned} R_{ig} &= \frac{|V_i|^2 P_i}{P_i^2 + Q_i^2}, \\ X_{ig} &= \frac{|V_i|^2 Q_i}{P_i^2 + Q_i^2}. \end{aligned} \tag{8}$$

NPF results vs direct LPF results

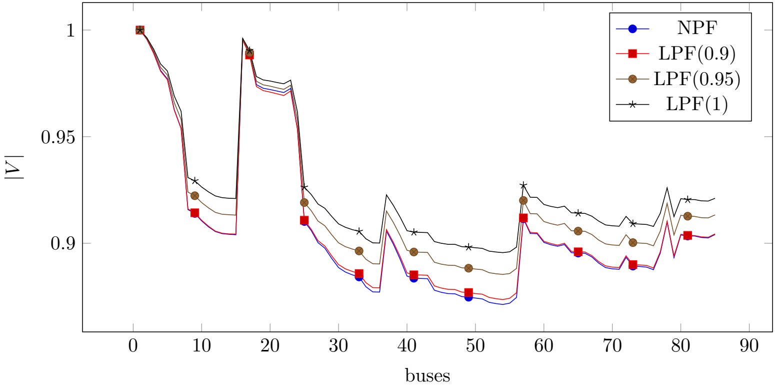


Figure: Voltage profile of the test case DCase85 for various $|\hat{V}_n|$.

NPF results and iterative LPF results

Table: The CPU time and the relative difference between NPF and iterative LPF.

Test cases	NPF($V^0 = 1.0$)		LPF($\lvert\hat{V}_n^0\rvert = 1$)		$\frac{\text{Time (NPF)}}{\text{Time (LPF)}}$	$\frac{\lvert\lvert V^N - V^L \rvert\rvert_2}{\lvert\lvert V^N \rvert\rvert_2}$
	Iter	Time(s)	Iter	Time(s)		
Dcase22	2	0.0201	4	0.0030	**6.72**	2.27×10^{-7}
Dcase33	3	0.0194	6	0.0033	**5.96**	4.36×10^{-7}
Dcase69	4	0.0205	6	0.0036	**5.76**	5.76×10^{-7}
Dcase85	3	0.0218	7	0.0040	**5.52**	1.70×10^{-6}
Dcase141	3	0.0237	6	0.0043	**5.50**	1.34×10^{-7}

Conclusion

- New variants of the Newton power flow method [5, 6] are more robust and faster than the existing versions of the Newton power flow method on both balanced and unbalanced networks.
- The performance of an OPF solution method can be improved by changing the mathematical formulation used to specify the OPF problem [7].
- New linear formulation of the original nonlinear power flow problem are proposed based on artificial ground buses [8].

Thank you!

Publications I

W. F. Tinney and C. E. Hart, "Power flow solution by Newton's method," *IEEE Transactions on Power Apparatus and systems*, no. 11, pp. 1449–1460, 1967.

J. E. Van Ness and J. H. Griffin, "Elimination methods for load-flow studies," *Transactions of the American Institute of Electrical Engineers. Part III: Power Apparatus and Systems*, vol. 80, no. 3, pp. 299–302, 1961.

H. W. Dommel, W. F. Tinney, and W. L. Powell, "Further developments in Newton's method for power system applications," *IEEE Winter Power Meeting, Conference Paper*, pp. CP 161–PWR New York, January 1970.

V. M. da Costa, N. Martins, and J. L. R. Pereira, "Developments in the Newton-Raphson power flow formulation based on current injections," *IEEE Transactions on Power Systems*, vol. 14, pp. 1320–1326, Nov 1999.

Publications II

B. Sereeter, K. Vuik, and C. Witteveen, "Newton power flow methods for unbalanced three-phase distribution networks," *Energies*, vol. 10, no. 10, p. 1658, 2017.

B. Sereeter, C. Vuik, and C. Witteveen, "On a comparison of Newton-Raphson solvers for power flow problems," *Journal of Computational and Applied Mathematics*, vol. 360, pp. 157–169, nov 2019.

B. Sereeter, C. Vuik, C. Witteveen, and P. Palensky, "Optimal power flow formulations and their impacts on the performance of solution methods," IEEE Power & Energy Society General Meeting, Aug 2019.

B. Sereeter, W. van Westering, C. Vuik, and C. Witteveen, "Linear power flow method improved with numerical analysis techniques applied to a very large network," *Energies*, vol. 12, no. 21, p. 4078, 2019.