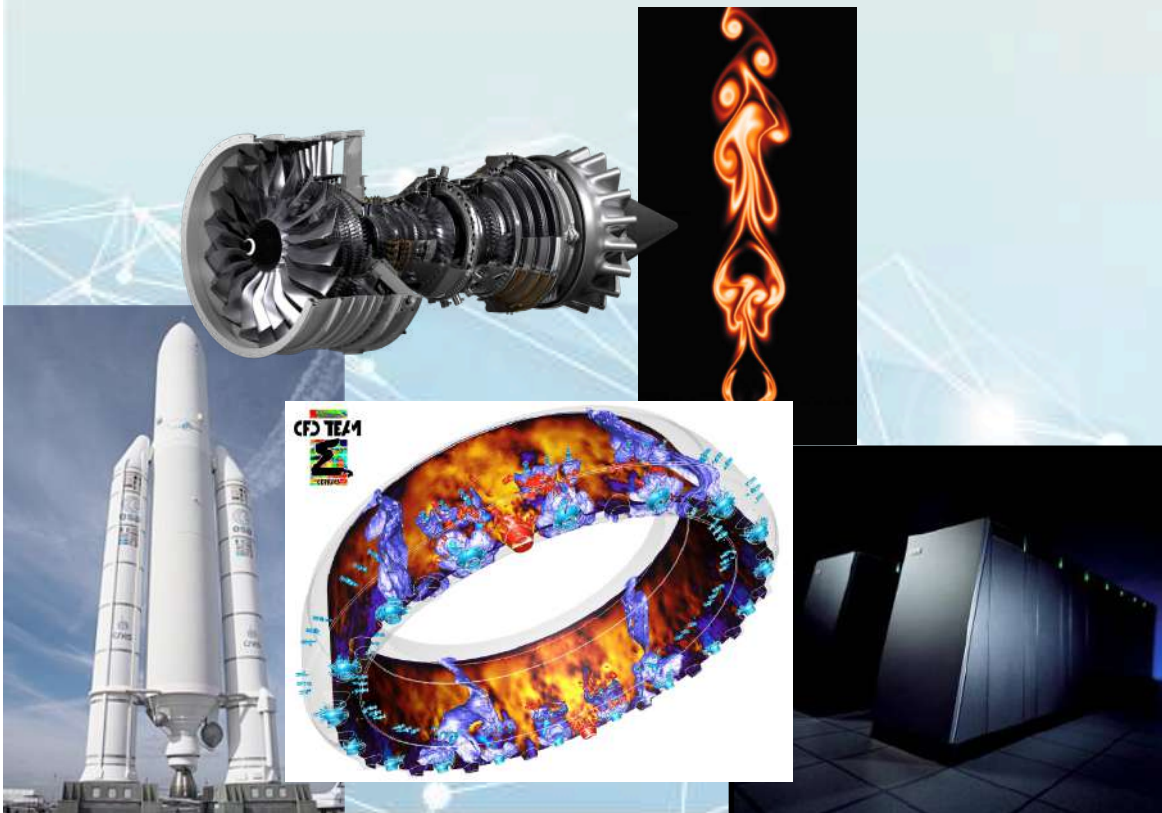


Numerical Simulation of Turbulent Combustion

B.Cuenot & the CFD Team





Combustion is (almost) everywhere ...



Furnaces



Transport



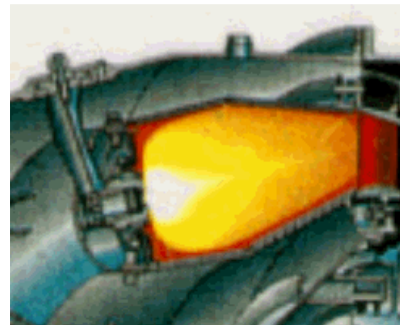
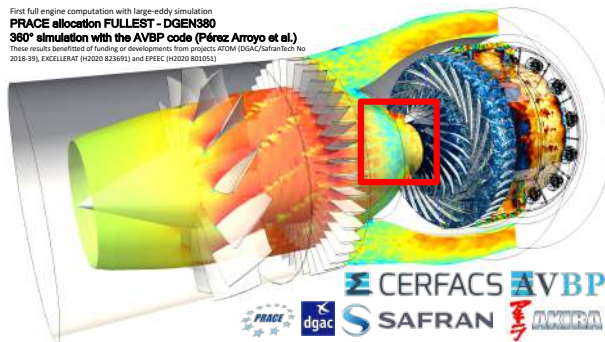
Fires



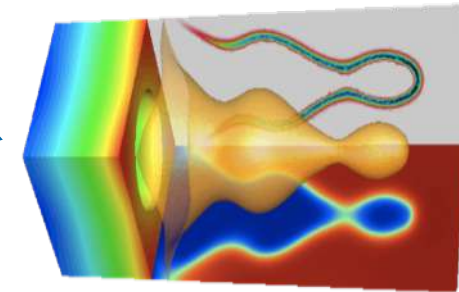
Why study Turbulent Combustion?

Practical combustion systems

*complex
geometry*



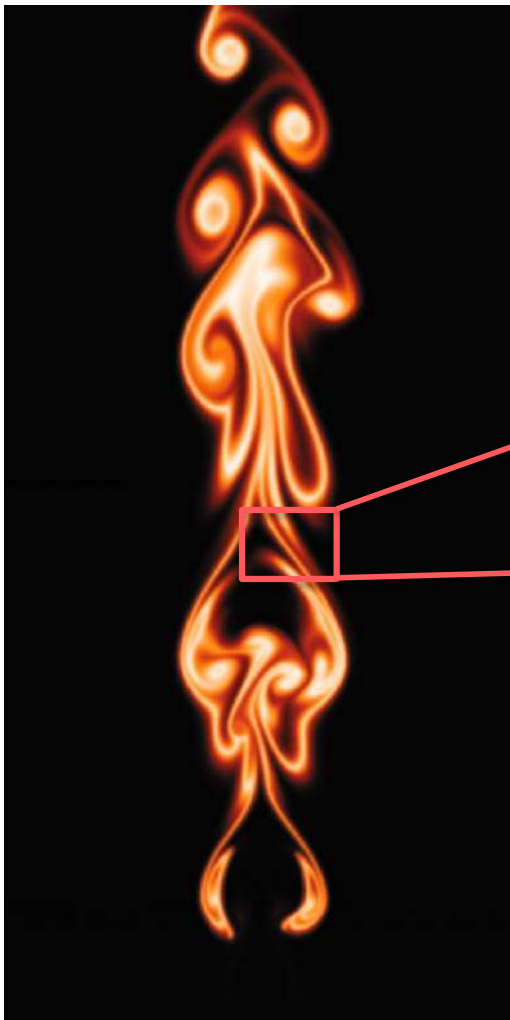
*kinetics
turb/chem
spray
emissions*



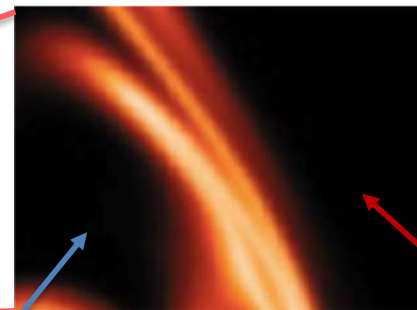
acoustics/comb

Advanced CFD and **Massively parallel** computer architectures offer a clear potential for time and cost reductions of the design process by providing **accurate predictions**

What is a flame?



A flame is the combination of thermochemical processes and transport (mixing)



Reactants

flame
zone

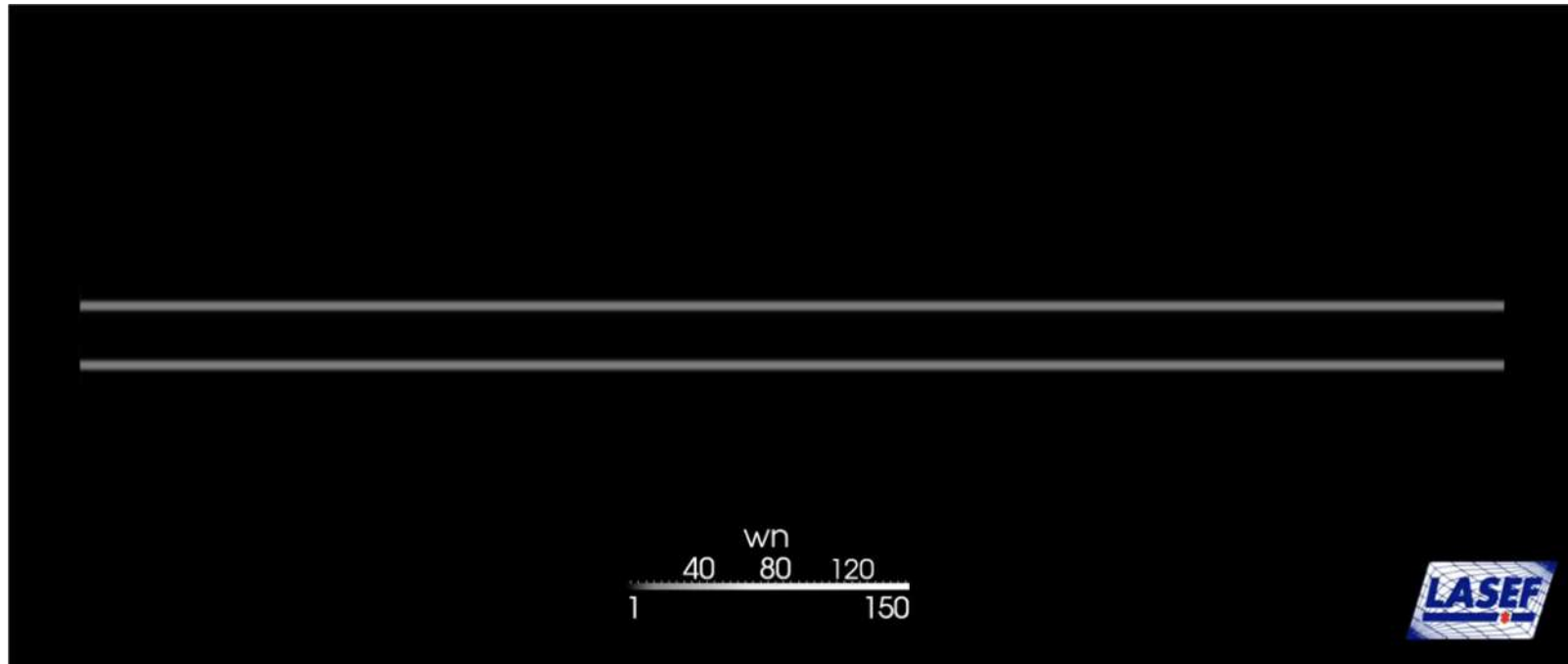
Products
+
Heat release

Flow model:

- variable density
- multi-species
- chemical source terms



What is turbulence?



Reynolds number UL / ν



What are the equations?

Navier-Stokes equations:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

- steady / unsteady
- compressible / low-mach

Species conservation equations

$$\frac{\partial(\rho Y_k)}{\partial t} + \frac{\partial(\rho(u_i + V_{k,i})Y_k)}{\partial x_i} = \dot{\omega}_k$$

- thermodynamic and transport properties
- chemical kinetics

Energy equation

$$\rho \frac{DE}{Dt} = -\frac{\partial q_i}{\partial x_i} + \frac{\partial}{\partial x_j}(\tau_{ij} u_i) - \frac{\partial}{\partial x_i}(P u_i) + \dot{\omega}_T + \dot{Q}$$

4 + N_{spec} conservation equations



What are the equations?

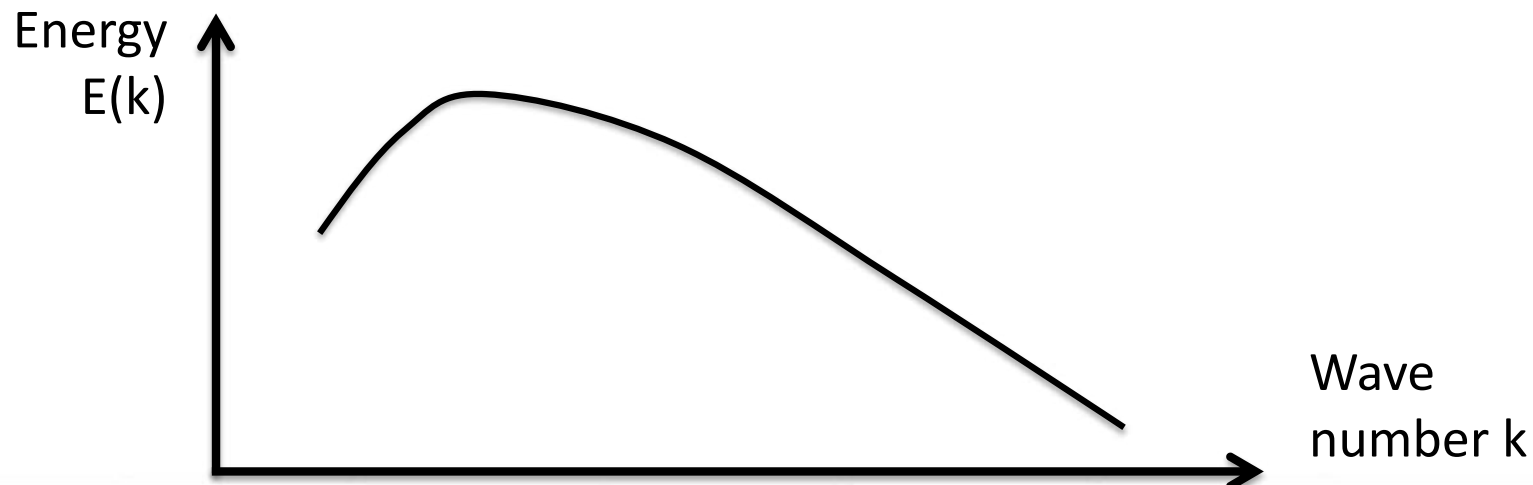
Navier-Stokes equations:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

Production / destruction terms of turbulence:

- Non-linear
- Large frequency spectrum

MODELLING !





What are the equations?

Navier-Stokes equations:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

- steady / unsteady
- compressible / low-mach

Species conservation equations

$$\frac{\partial(\rho Y_k)}{\partial t} + \frac{\partial(\rho(u_i + V_{k,i})Y_k)}{\partial x_i} = \dot{\omega}_k$$

- thermodynamic and transport properties
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Energy equation

$$\rho \frac{DE}{Dt} = -\frac{\partial q_i}{\partial x_i} + \frac{\partial}{\partial x_j}(\tau_{ij} u_i) - \frac{\partial}{\partial x_i}(P u_i) + \dot{\omega}_T + \dot{Q}$$

Combustion terms

The progress rate of the j^{th} reaction is given by:

$$Q_j = K_j^f \prod_{k=1}^n \left[\frac{\rho Y_k}{W_k} \right]^{v'_{kj}} - K_j^b \prod_{k=1}^n \left[\frac{\rho Y_k}{W_k} \right]^{v''_{kj}}$$

Reverse rate of reaction
Forward rate of reaction

The essential problem of chemists is to provide these two rates. They usually rely on the Arrhenius law: and equilibrium constants:

$$K_j^f = A_j^f T^{\beta_j} \exp(-E_j / RT)$$

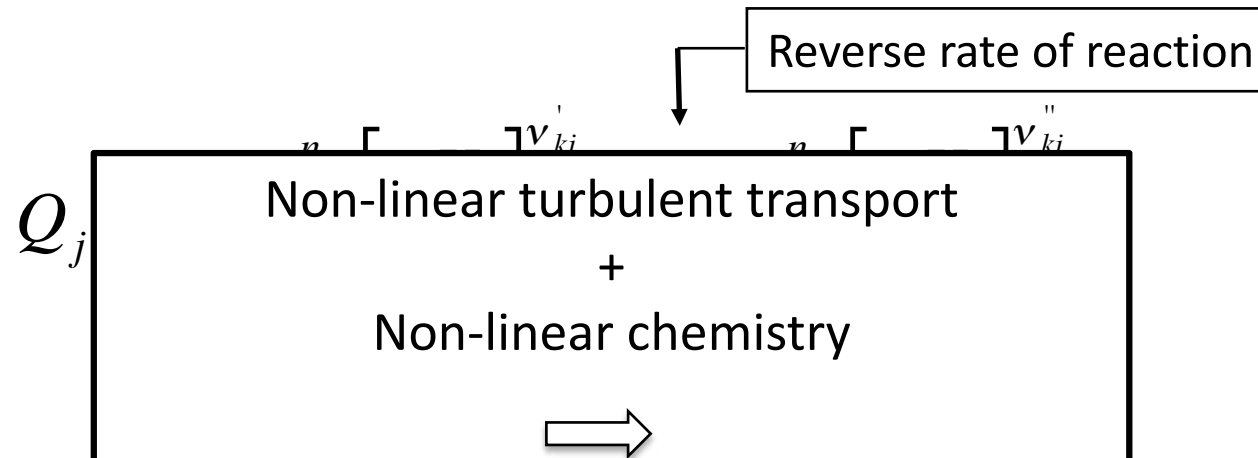
Collision frequency:
 - Pre-exponential Cst
 - Temp. modulation

Activation energy

$$K_j^r = \frac{K_j^f}{(P_a / RT)^{\sum_{k=1}^n (v'_{kj} - v''_{kj})}} \exp\left(\frac{\Delta S_j^0}{R} - \frac{\Delta H_j^0}{RT}\right)$$

Entropy and enthalpy changes of j^{th} reaction (from the thermodynamics table)

The progress rate of the j^{th} reaction is given by:



The essential problem is the complex chemistry / turbulence interaction s. They usually rely on the Arrhenius law:

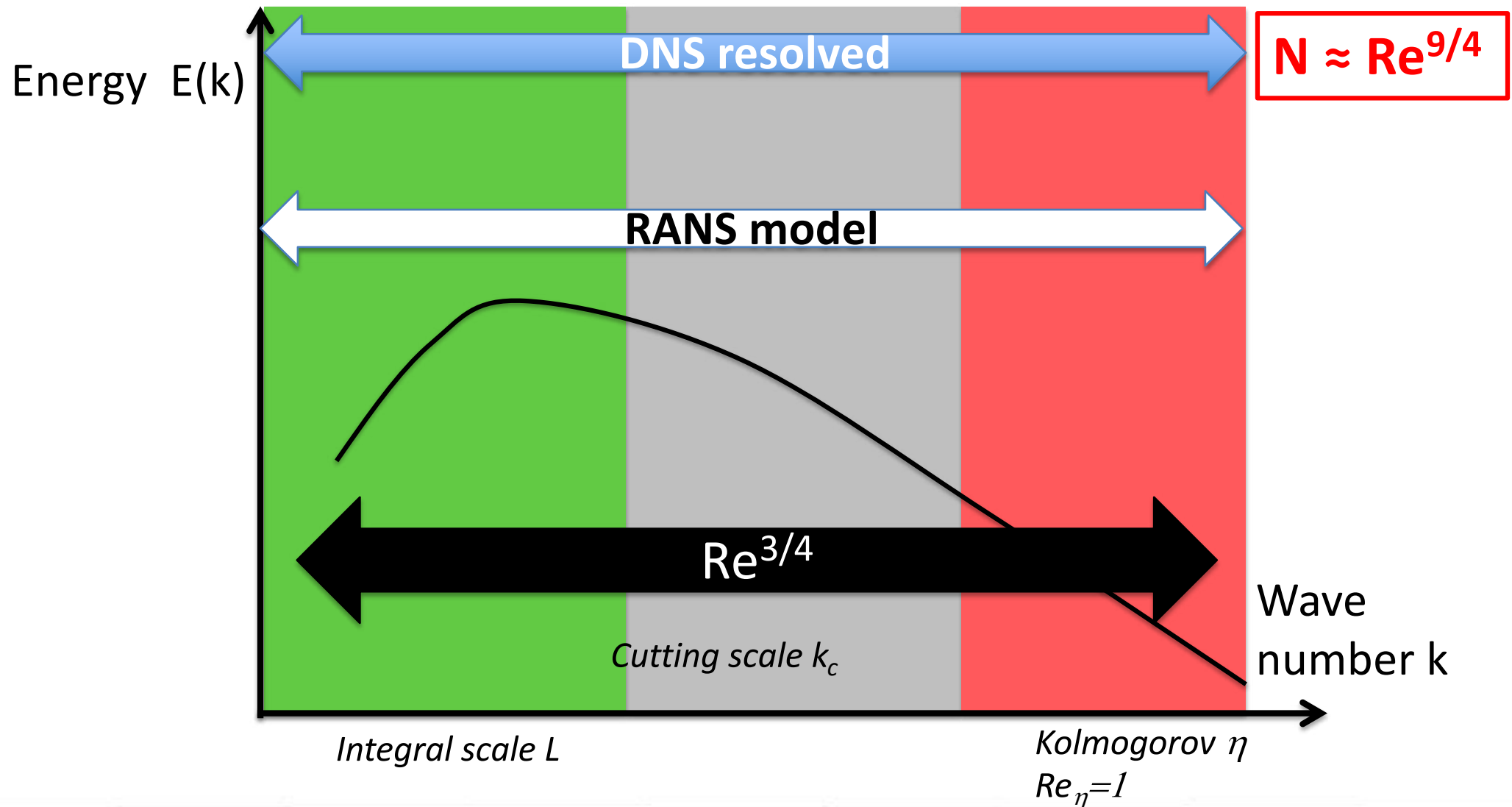
$$K_j^f = A_j^f T^{\beta_j} \exp\left(\frac{\Delta S_j^0}{R} - \frac{\Delta H_j^0}{RT}\right) \left(\frac{P_a}{RT}\right)^{\sum_{k=1}^n (v'_{kj} - v''_{kj})}$$

Collision frequency:
- Pre-exponential Cst
- Temp. modulation

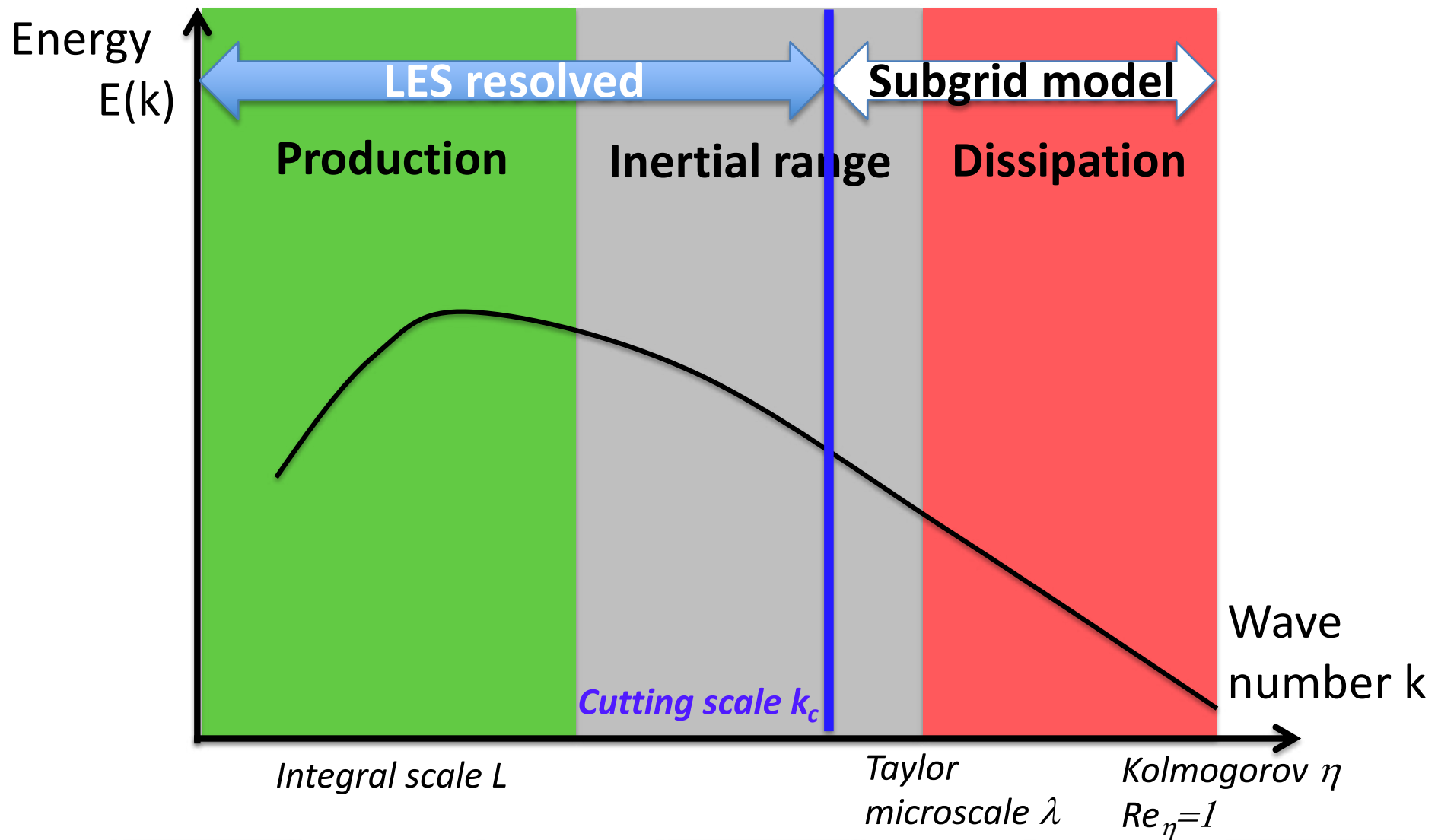
Activation energy

Entropy and enthalpy changes of j^{th} reaction (from the thermodynamics table)

Numerical Strategies to solve turbulence



Numerical Strategies to solve turbulence : Large Eddy Simulation





Modeling turbulence in LES

The **filtered** equation

$$\frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} \tilde{u}_i \tilde{u}_j) + \frac{\partial \bar{p}}{\partial x_j} = \frac{\partial}{\partial x_i} [\bar{\tau}_{ij} - \bar{\rho} (\overline{u_i u_j} - \tilde{u}_i \tilde{u}_j)]$$

$$\tau_{ij} = \overline{u_i u_j} - \tilde{u}_i \tilde{u}_j = \frac{\delta_{ij}}{3} \tau_{kk} - 2 \nu_t \left(\tilde{S}_{ij} - \frac{\delta_{ij}}{3} \tilde{S}_{kk} \right)$$

$$\nu_t = (C_s \Delta)^2 \sqrt{2 \tilde{S}_{ij} \tilde{S}_{ij}}$$

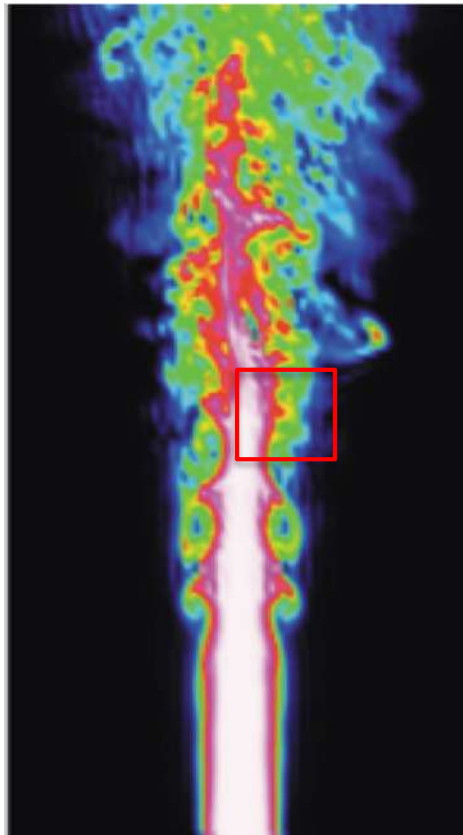
- Smagorinsky model
- Germano dynamic model
- WALE model
- Structural models
- ...and others



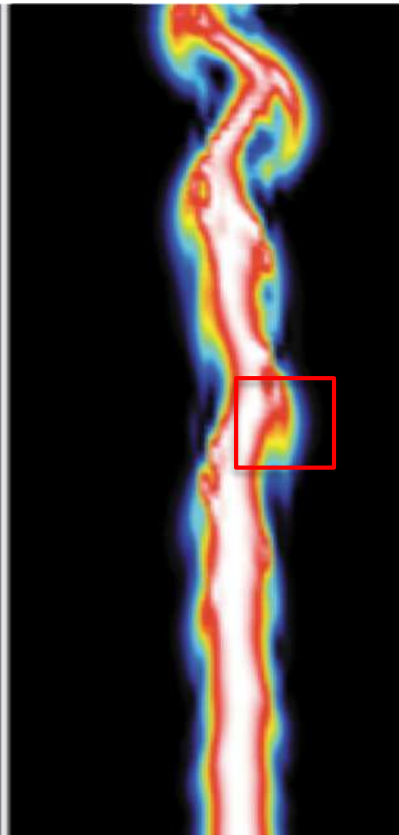
Modeling combustion in LES

The **filtered** source term

DNS



LES



Two major approaches

- PDF-based

$$\tilde{\Phi} = \int_{c^*} \int_{Z^*} \Phi(c^*, Z^*) P(c^*) P(Z^*) dc^* dZ^*$$



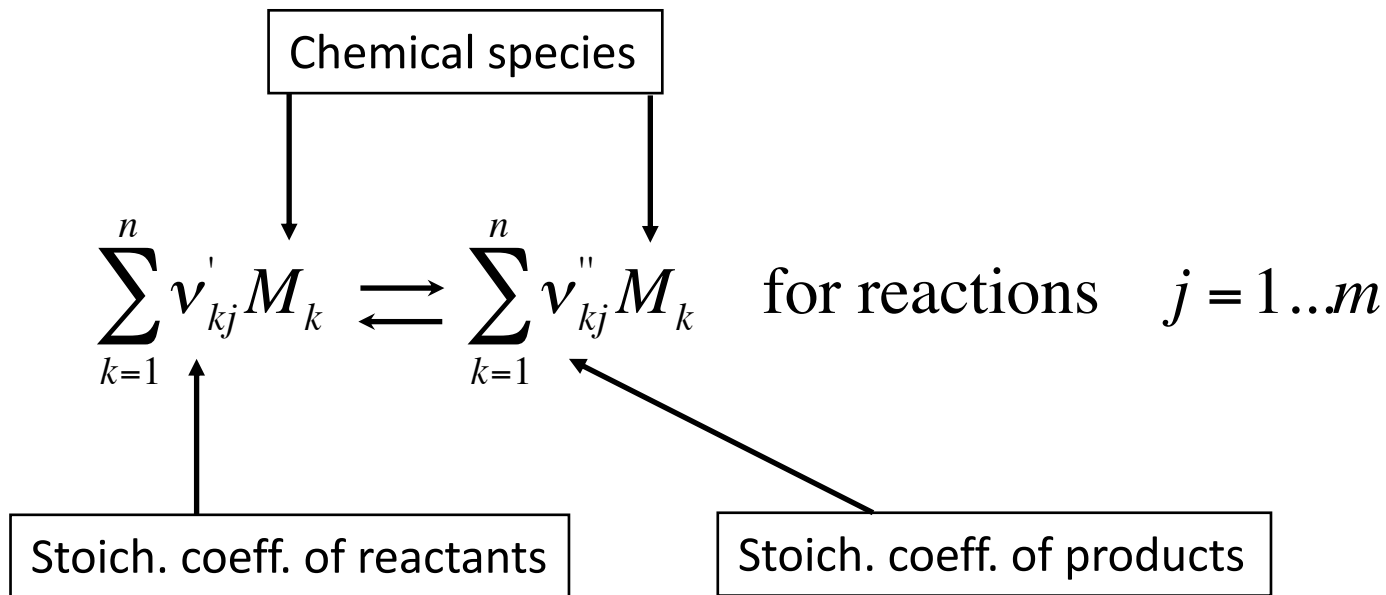
Given flame structure

- Thickened flame

Resolved flame structure

Loss of subgrid-scale wrinkling → longer flame

Stoichiometric relationships for an arbitrarily complex set of m reactions involving n species, may be written:



Nb	Reaction
1	$\text{H}_2 + \text{O}_2 \rightleftharpoons 2\text{OH}$
2	$\text{OH} + \text{H}_2 \rightleftharpoons \text{H}_2\text{O} + \text{H}$
3	$\text{H} + \text{O}_2 \rightleftharpoons \text{OH} + \text{O}$
4	$\text{O} + \text{H}_2 \rightleftharpoons \text{OH} + \text{H}$
5 ^a	$\text{H} + \text{O}_2 + \text{M} \rightleftharpoons \text{HO}_2 + \text{M}$
6	$\text{H} + \text{O}_2 + \text{O}_2 \rightleftharpoons \text{HO}_2 + \text{O}_2$
7	$\text{H} + \text{O}_2 + \text{N}_2 \rightleftharpoons \text{HO}_2 + \text{N}_2$
8	$\text{OH} + \text{HO}_2 \rightleftharpoons \text{H}_2\text{O} + \text{O}_2$
9	$\text{H} + \text{HO}_2 \rightleftharpoons 2\text{OH}$
10	$\text{O} + \text{HO}_2 \rightleftharpoons \text{O}_2 + \text{OH}$
11	$2\text{OH} \rightleftharpoons \text{O} + \text{H}_2\text{O}$
12 ^b	$\text{H}_2 + \text{M} \rightleftharpoons \text{H} + \text{H} + \text{M}$
13	$\text{O}_2 + \text{M} \rightleftharpoons \text{O} + \text{O} + \text{M}$
14 ^c	$\text{H} + \text{OH} + \text{M} \rightleftharpoons \text{H}_2\text{O} + \text{M}$
15	$\text{H} + \text{HO}_2 \rightleftharpoons \text{H}_2 + \text{O}_2$
16	$\text{HO}_2 + \text{HO}_2 \rightleftharpoons \text{H}_2\text{O}_2 + \text{O}_2$
17	$\text{H}_2\text{O}_2 + \text{M} \rightleftharpoons \text{OH} + \text{OH} + \text{M}$
18	$\text{H}_2\text{O}_2 + \text{H} \rightleftharpoons \text{HO}_2 + \text{H}$
19	$\text{H}_2\text{O}_2 + \text{OH} \rightleftharpoons \text{H}_2\text{O} + \text{HO}_2$

Reduced chemical schemes

Accuracy

- Detailed schemes (GRImech~500 reactions) are produced from very large size schemes developed by chemists and are able to accurately describe oxidation of most hydrocarbons
- A first reduction step leads to skeletal mechanisms (~50-100 reactions) that keep most of the performances of detailed schemes
- A second reduction step (QSS) leads to analytical schemes (~10-30 reactions) having reduced but still reasonable performances
- Finally, optimally fitted global or semi-global schemes (1 to 4 steps) often require on-line adjustment of constants with P , Φ , etc

CPU cost

In addition, complex fuels are simplified and described with surrogate fuels

Component	Surrogate 1		Surrogate 2	
	mole [%]	vol. [%]	mole [%]	vol. [%]
<i>n</i> -heptane	22.0	23.5	66.8	74.1
iso-octane	44.8	53.6	0.4	0.5
benzene	13.0	10.0	24.8	20.0
toluene	20.2	12.9	7.9	5.3
Avg. formula	$C_{7.32}H_{14.0}$		$C_{6.75}H_{12.9}$	



AVBP – An unstructured LES solver

developed by CERFACS

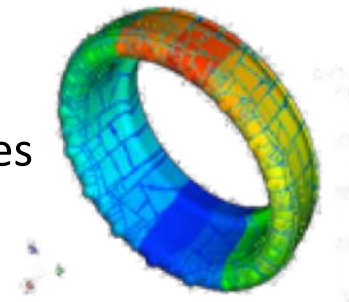
- External, internal flows
- Fully compressible turbulent reacting flows (ideal & real gas thermo.)
- DNS / LES approach

- Unstructured hexaedral, tetraedral, prisms & hybrid meshes
- Massively parallel, SPMD approach
- Explicit in time

- **Centered schemes**

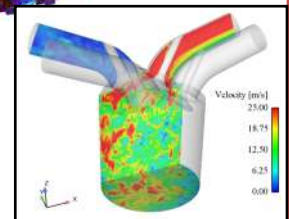
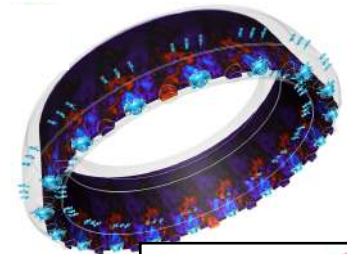
Finite Volumes / Finite Elements (2nd/3rd order^a)

- SGS models : Smagorinsky(dynamic)/WALE^b
- NSCBC^c boundary cond. + wall laws
- Reduced^d or tabulated^e chemical kinetics
- Thickened flame turb. combustion model (TFLES)^f
- Multi-phase solvers (Lagrangian & Eulerian)

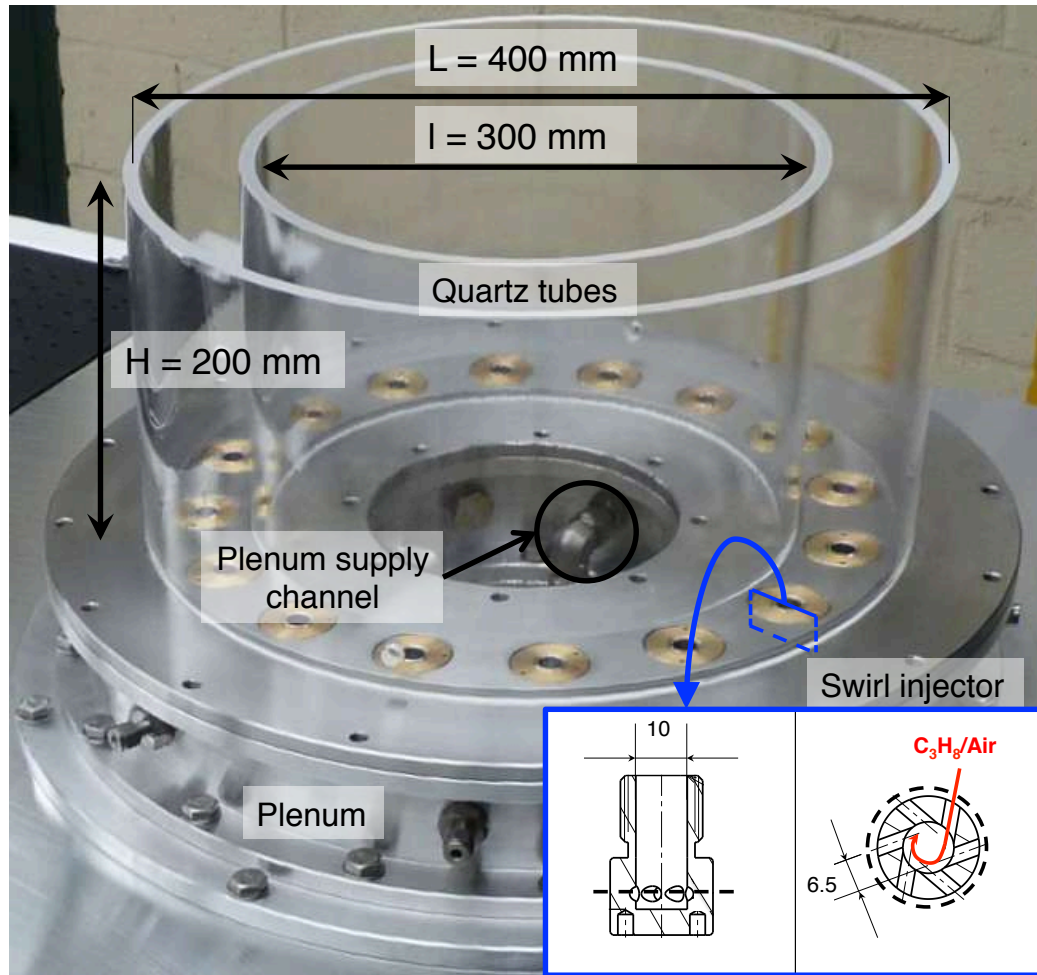


Applications

- ◆ Gas turbines
- ◆ Aeronautical engines
- ◆ Piston engines
- ◆ Statoreactor
- ◆ Rocket engines
- ◆ Furnaces
- ◆ Heat exchangers



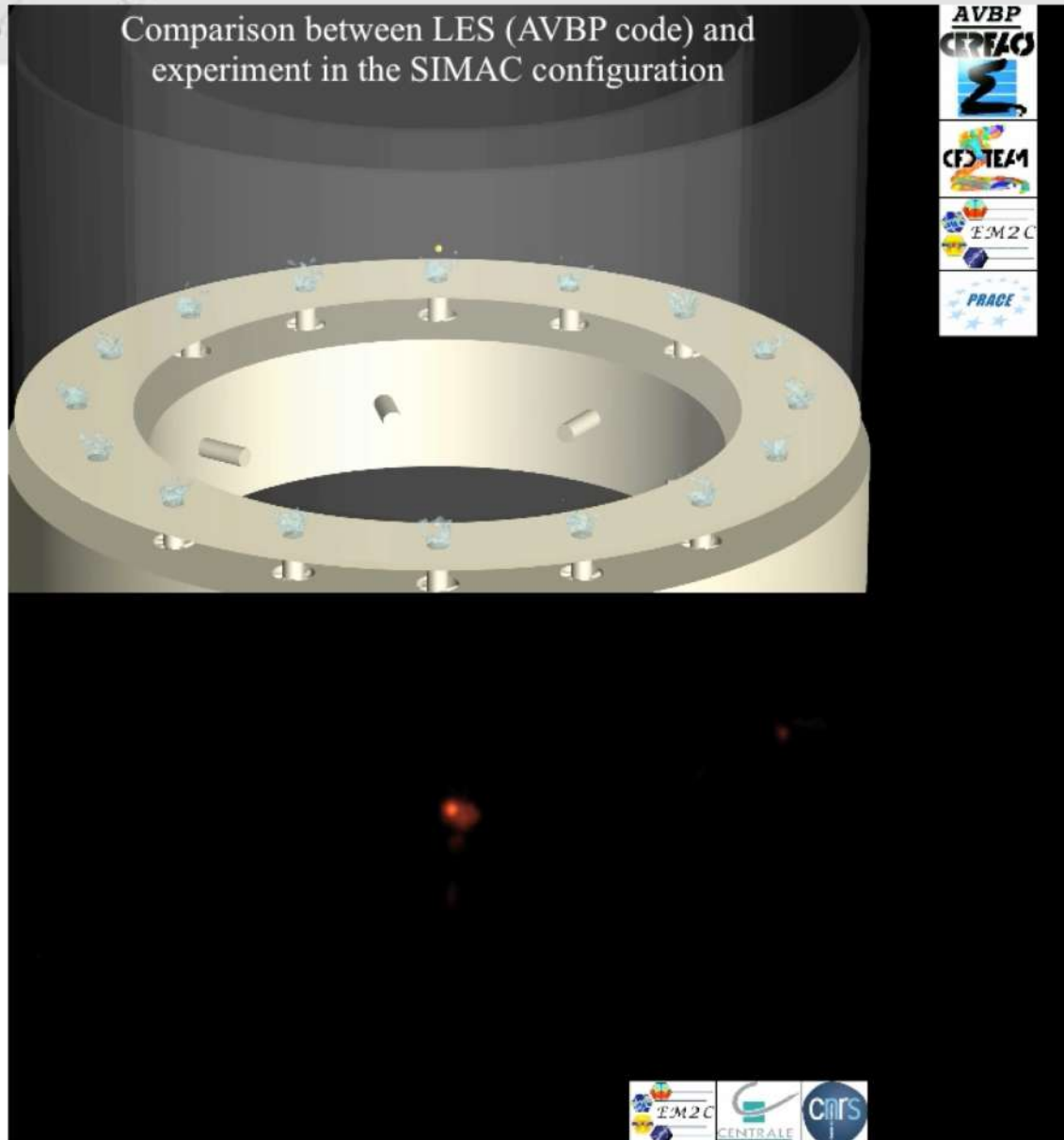
Ignition of the MICCA burner of EM2C



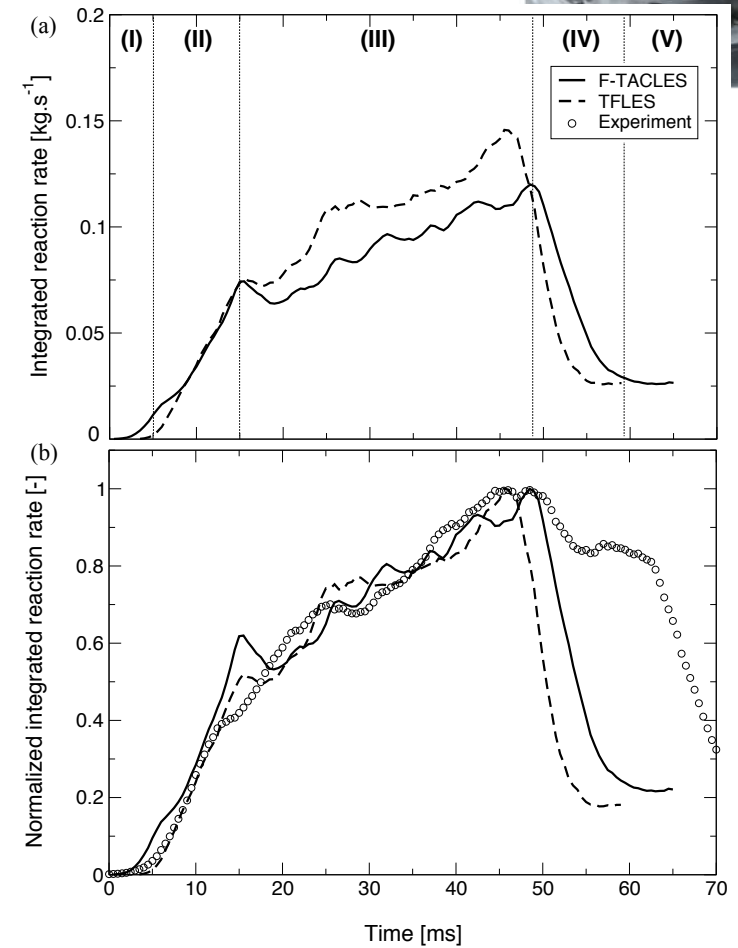
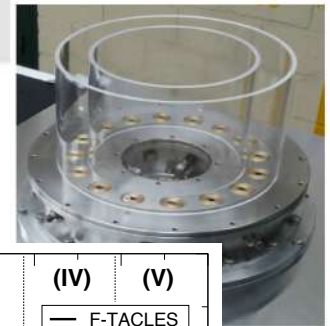
MICCA (EM2C) (*Durox et al*)

- Annular chamber
- 16 swirled injectors
- propane
- transparent walls

Ignition of the MICCA burner of EM2C

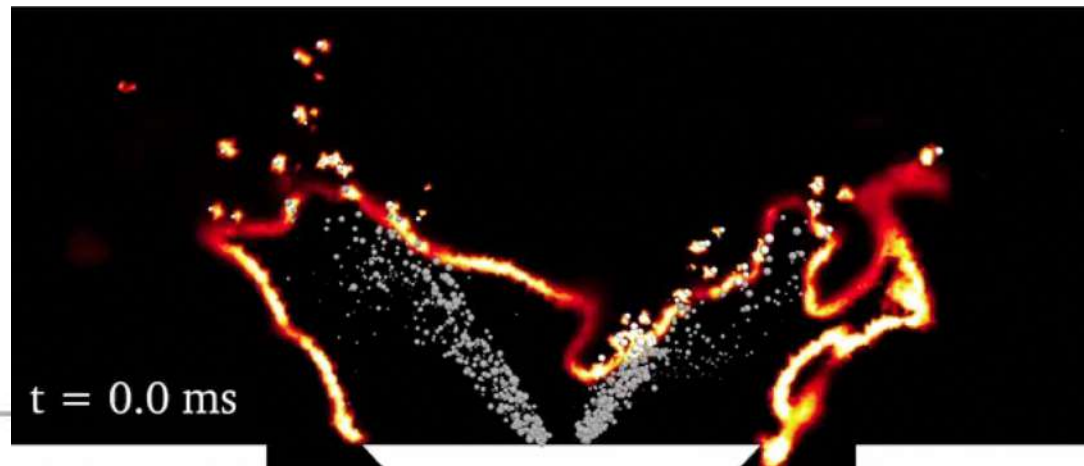
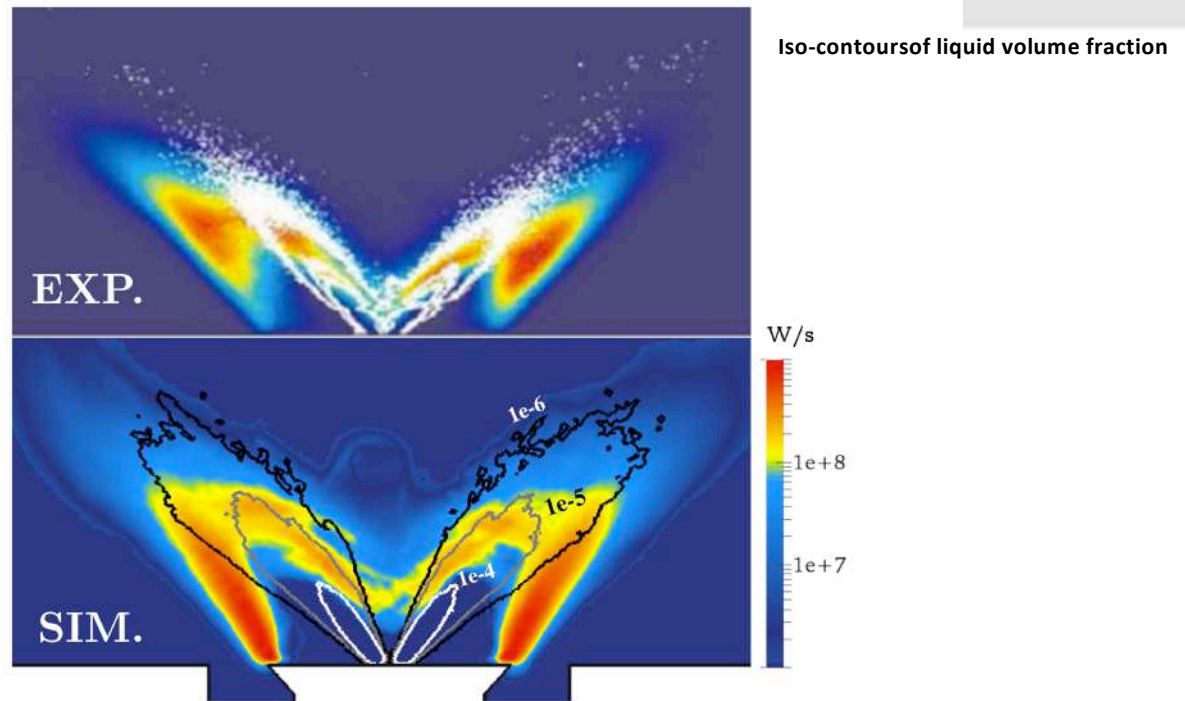
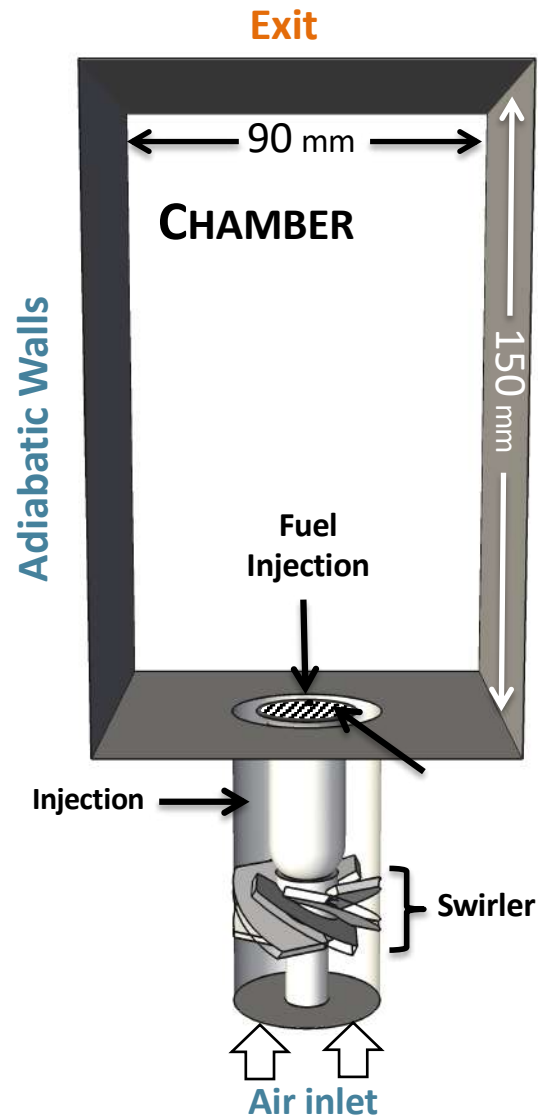


Philip et al, ASME 2013



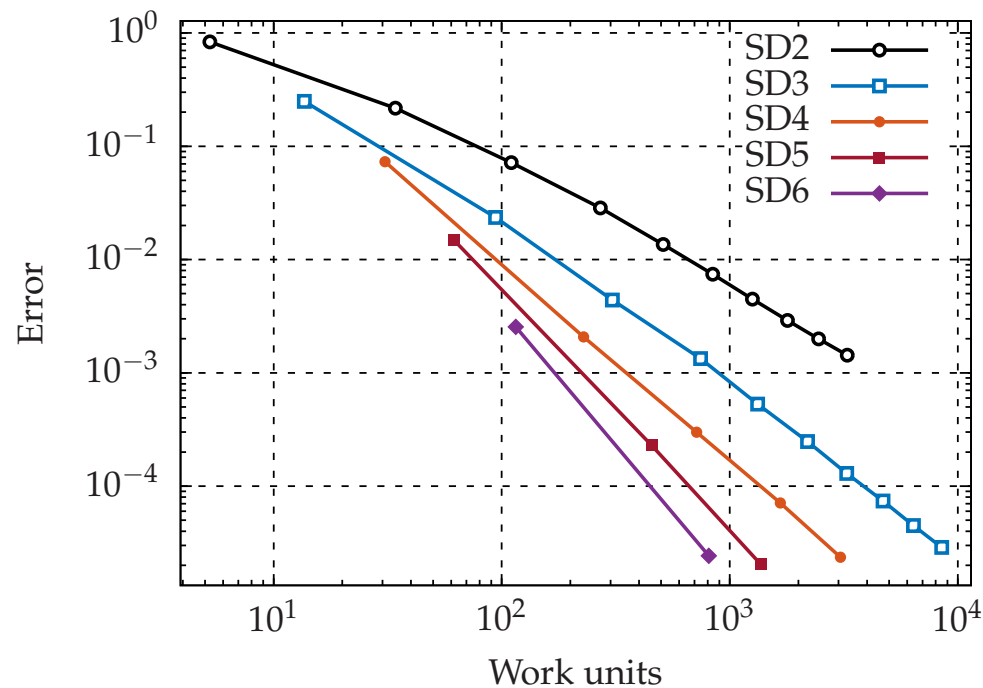
Adding two-phase flow : liquid fuel

Experiment of Cambridge university



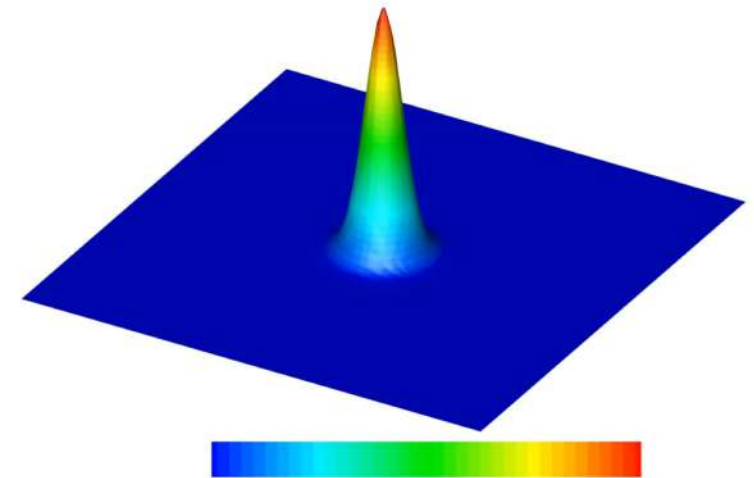
Other numerical approaches

High order methods

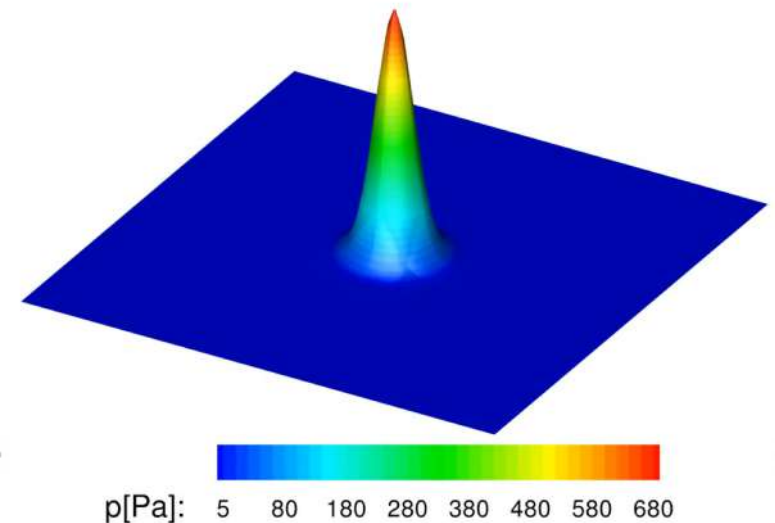


For a given precision, high order methods are computationally efficient

Low order
Time = 0.000T
SD, 2nd order



High order
Time = 0.000T
SD, 5th order





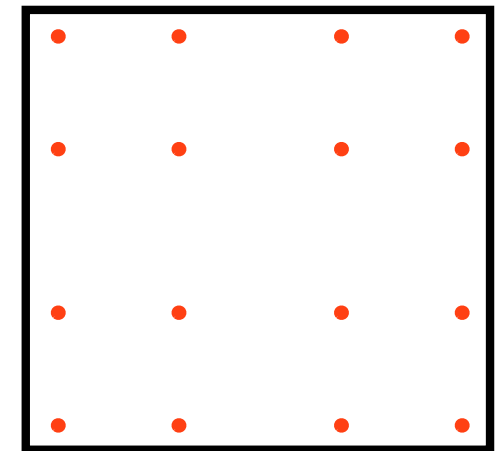
Spectral discontinuous approach

Assumptions:

- Unknowns are represented **locally** (inside a cell) as a **polynomial of degree p**
- **No continuity** is assumed **at the interface between two adjacent cells**
- Compute an **interface flux polynomial** using (approximated) **Riemann solvers** as in **Finite Volume Method**

Several techniques:

- Solve the *weak* form of the equation locally
⇔ **Discontinuous Galerkin method**
- Solve the *strong* form of the equation locally
⇔ **Spectral Difference method**



One high-order 2D cell



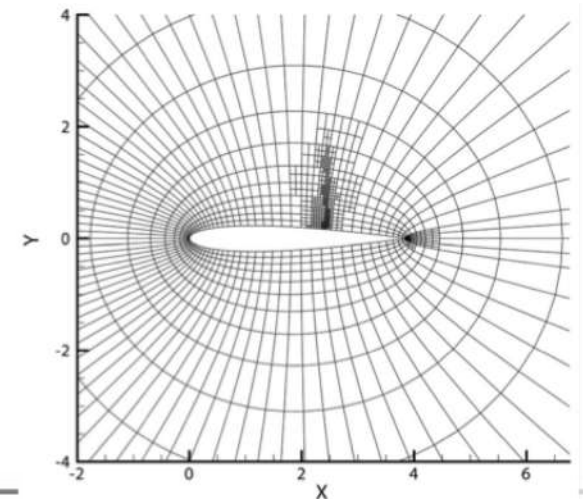
Spectral discontinuous approach

Promising techniques for LES and DNS

- **Accuracy:** control by the polynomial degree p chosen by the user
- **Local treatment:**
 - Algorithm can easily be written in term of **matrix/vector product** \Leftrightarrow **Good vectorization**
 - **Very compact stencil** \Leftrightarrow **High Parallel Efficiency**
- **Compatible with Unstructured grids**

hp-adaptation

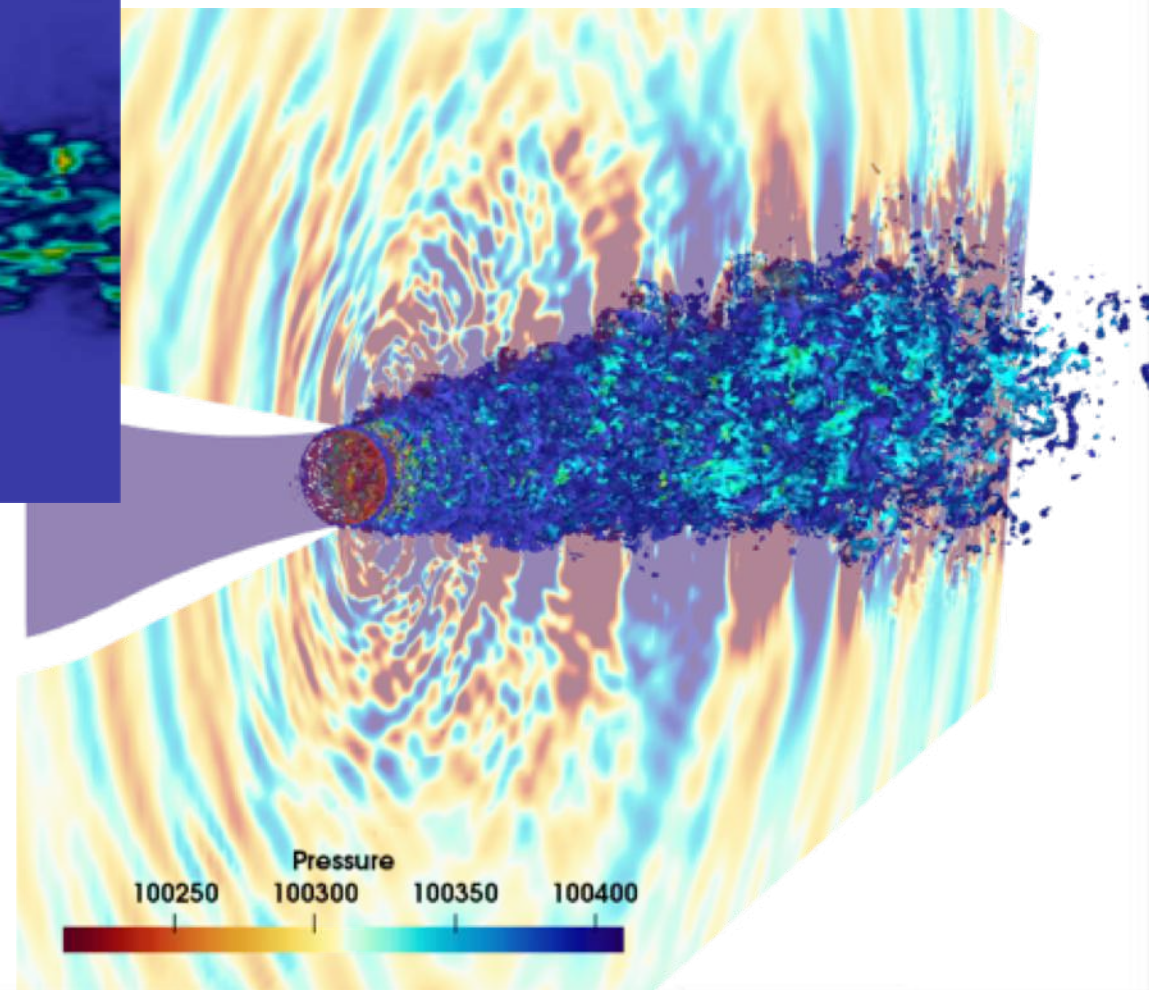
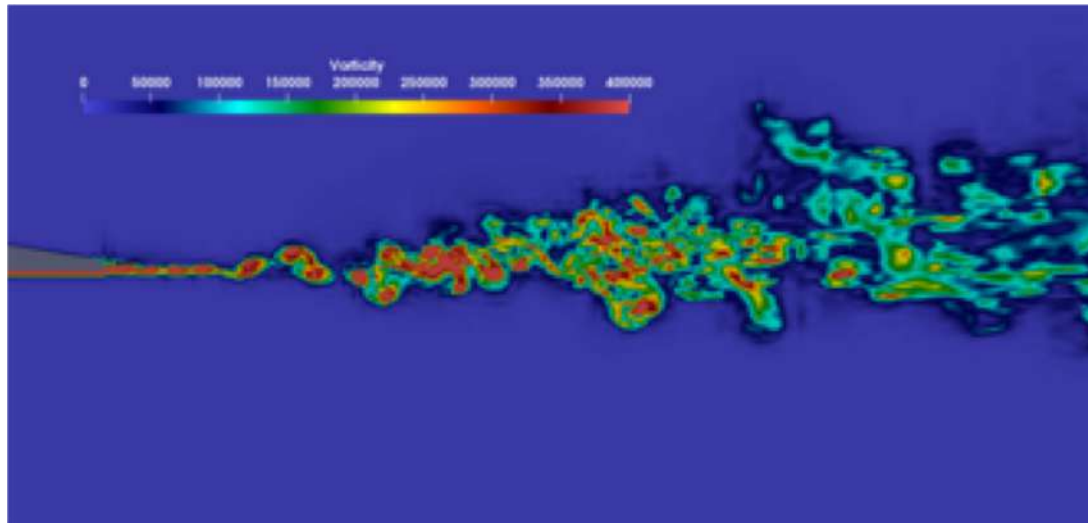
- Change **local refinement** (size h of the cell)
- Change **solution accuracy** (degree p of the polynomial representation)





Spectral discontinuous approach

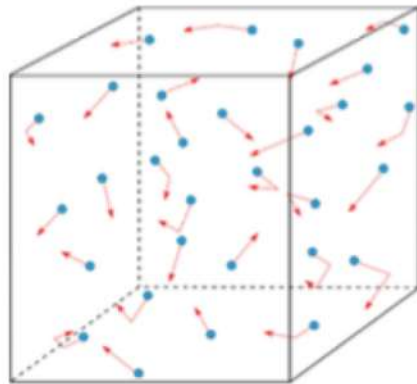
Example : jet noise simulation





Lattice Boltzman Method (LBM)

Can we avoid meshing complex geometries ?



Lattice Boltzmann equation

$$\frac{\partial f}{\partial t} + \xi_\beta \frac{\partial f}{\partial x_\beta} + F_\beta \frac{\partial f}{\partial \xi_\beta} = \Omega(f) \quad \nearrow \quad -\frac{1}{\tau}(f - f^0) \quad \text{BGK}$$

Maxwell-Boltzmann distribution function

$$f^{(0)} = \frac{\rho}{(2\pi c_T^2)^{D/2}} \exp \left(\frac{-(\vec{\xi} - \vec{u})^2}{2c_T^2} \right)$$

Continuum-statistical variables

$$\rho(x, t) \equiv mn(x, t) = m \int f dv$$

$$\rho u(x, t) = m \int f v dv$$

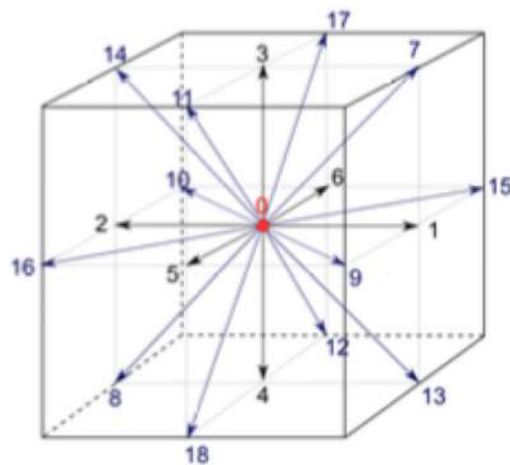
$$\rho e(x, t) = \frac{1}{2} m \int f \underbrace{|v - u|}_c dv$$

Boivin et al. , M2P2



Lattice Boltzman Method (LBM)

Can we avoid meshing complex geometries ?



Lattice Boltzmann equation

$$f_i(\mathbf{x} + \mathbf{c}_i \delta t, t + \delta t) - f_i(\mathbf{x}, t) = -\frac{1}{\tau} [f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)]$$

Streaming

Collision

Maxwell-Boltzmann distribution function

$$f_i^{eq} = \rho w_i \left[1 + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}^2}{2c_s^2} \right]$$

Boivin et al. , M2P2



Lattice Boltzman Method (LBM)

Can we avoid meshing complex geometries ?

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) &= 0 \\ \frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u u) &= -\nabla p + \nabla \cdot [\rho \nu (\nabla u + (\nabla u)^T)]\end{aligned}$$

LBM

$$\frac{\partial T}{\partial t} + u_\alpha \frac{\partial}{\partial x_\alpha} T = \frac{1}{\rho} \frac{\partial}{\partial x_\alpha} \left(\rho D_T \frac{\partial T}{\partial x_\alpha} \right) + \frac{\omega_h}{\rho c_p}$$

+

$$\frac{\partial Y_k}{\partial t} + u_\alpha \frac{\partial}{\partial x_\alpha} Y_k = \frac{1}{\rho} \frac{\partial}{\partial x_\alpha} \left(\rho D_k \frac{\partial Y_k}{\partial x_\alpha} \right) + \frac{\omega_k}{\rho}$$

**Finite
differences**

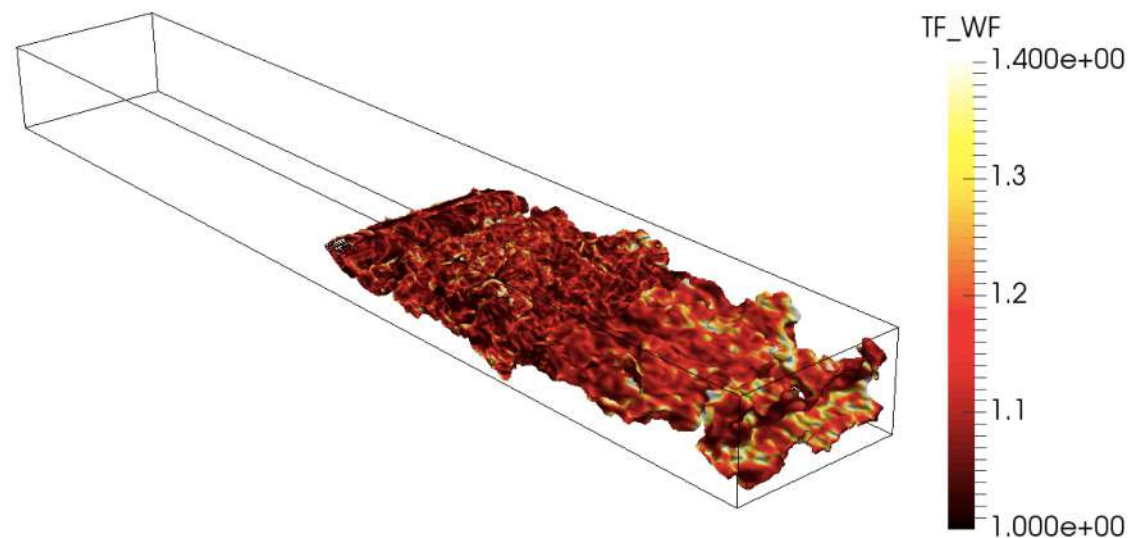
Boivin et al., M2P2



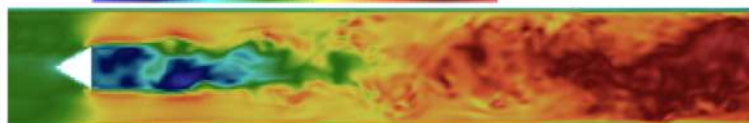
Lattice Boltzman Method (LBM)

VOLVO case

Sjunnesson et al., 1991

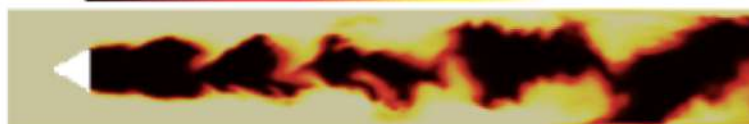


-1.943e+01 3.2 26 49 7.122e+01



Axial Velocity

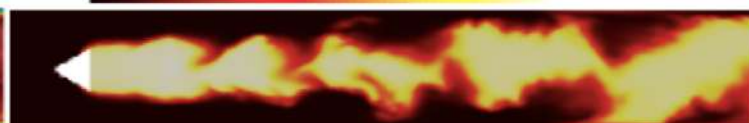
0.000e+00 0.011 0.021 0.032 4.201e-02



50

C₃H₈

2.484e+02 681 1113 1546 1.978e+03



Temperature

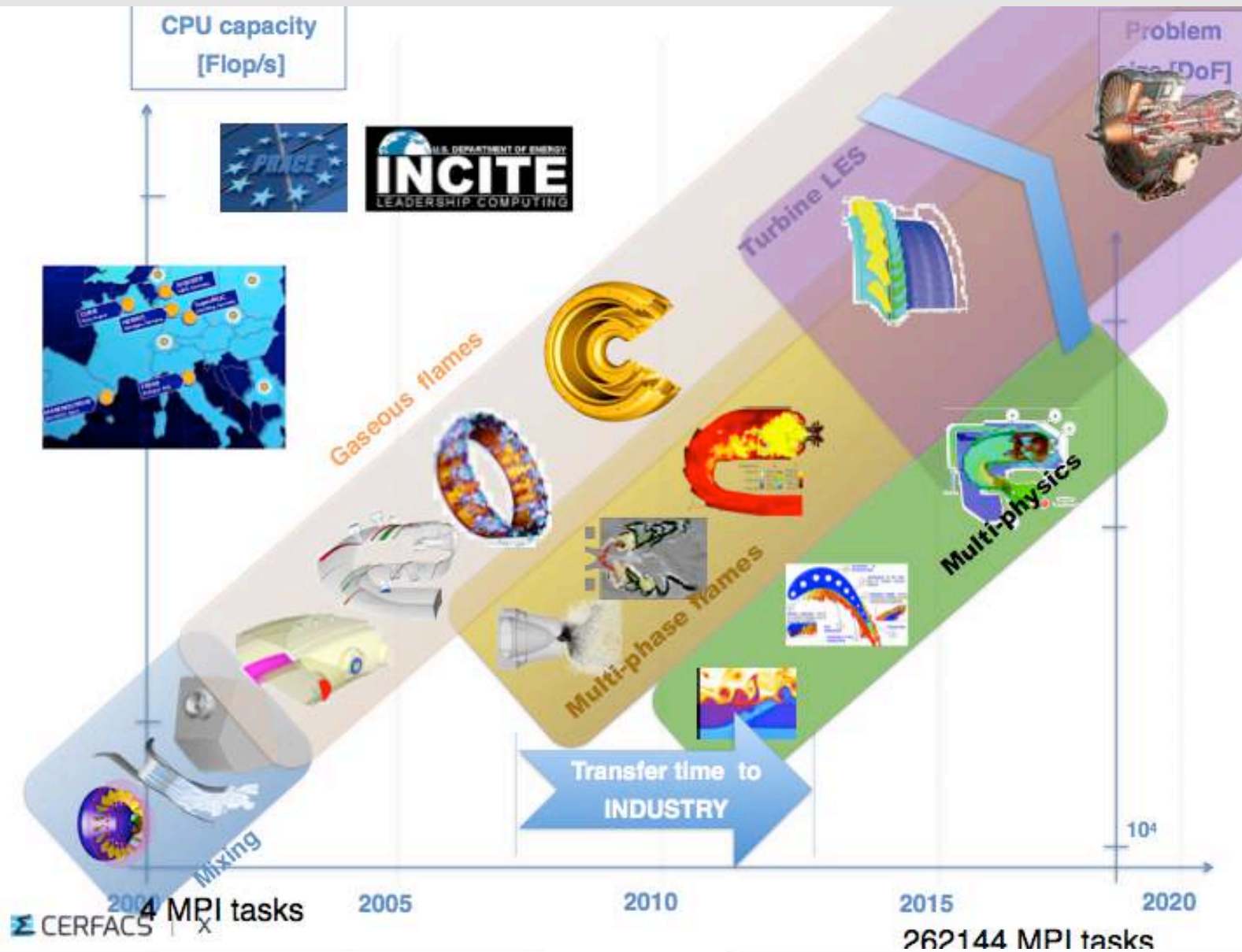
0.000e+00 6.1e+8 1.2e+9 1.8e+9 2.459e+09



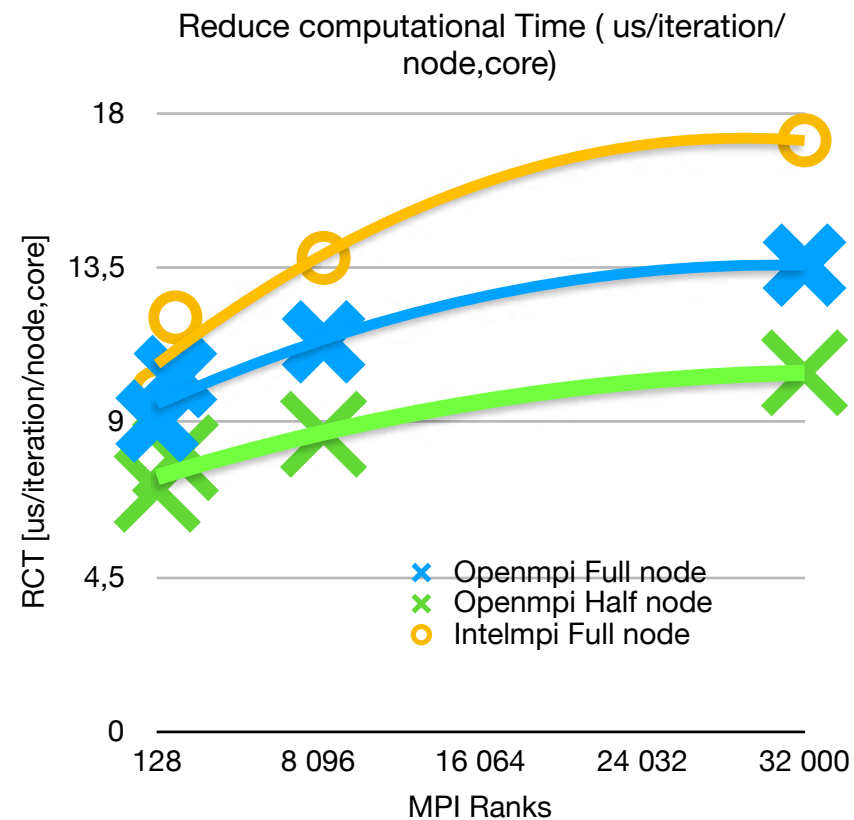
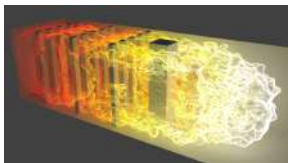
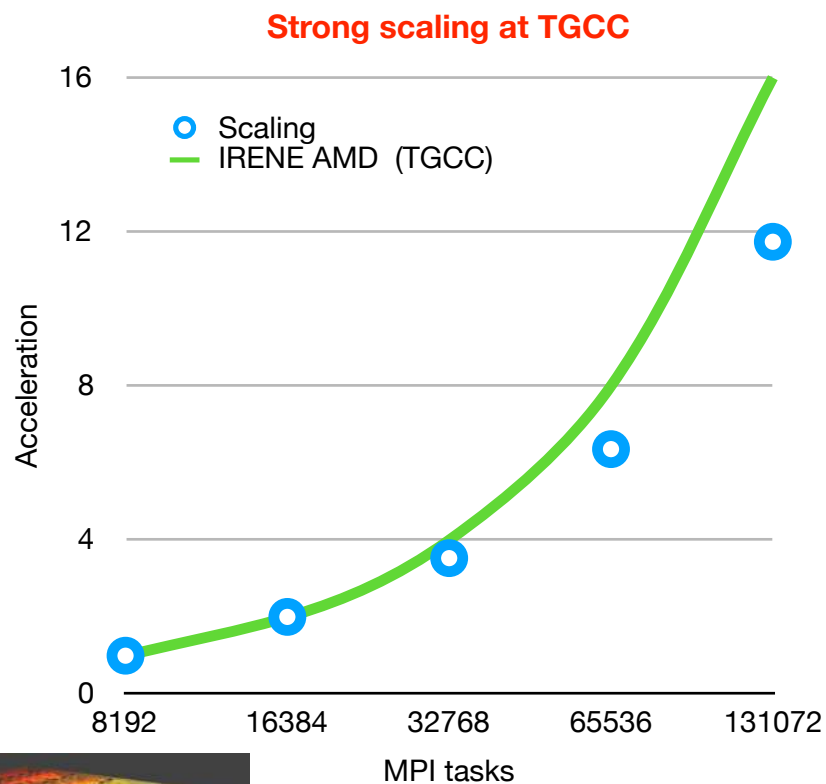
Heat Release Rate

Boivin et al. , M2P2

High Performance Computing

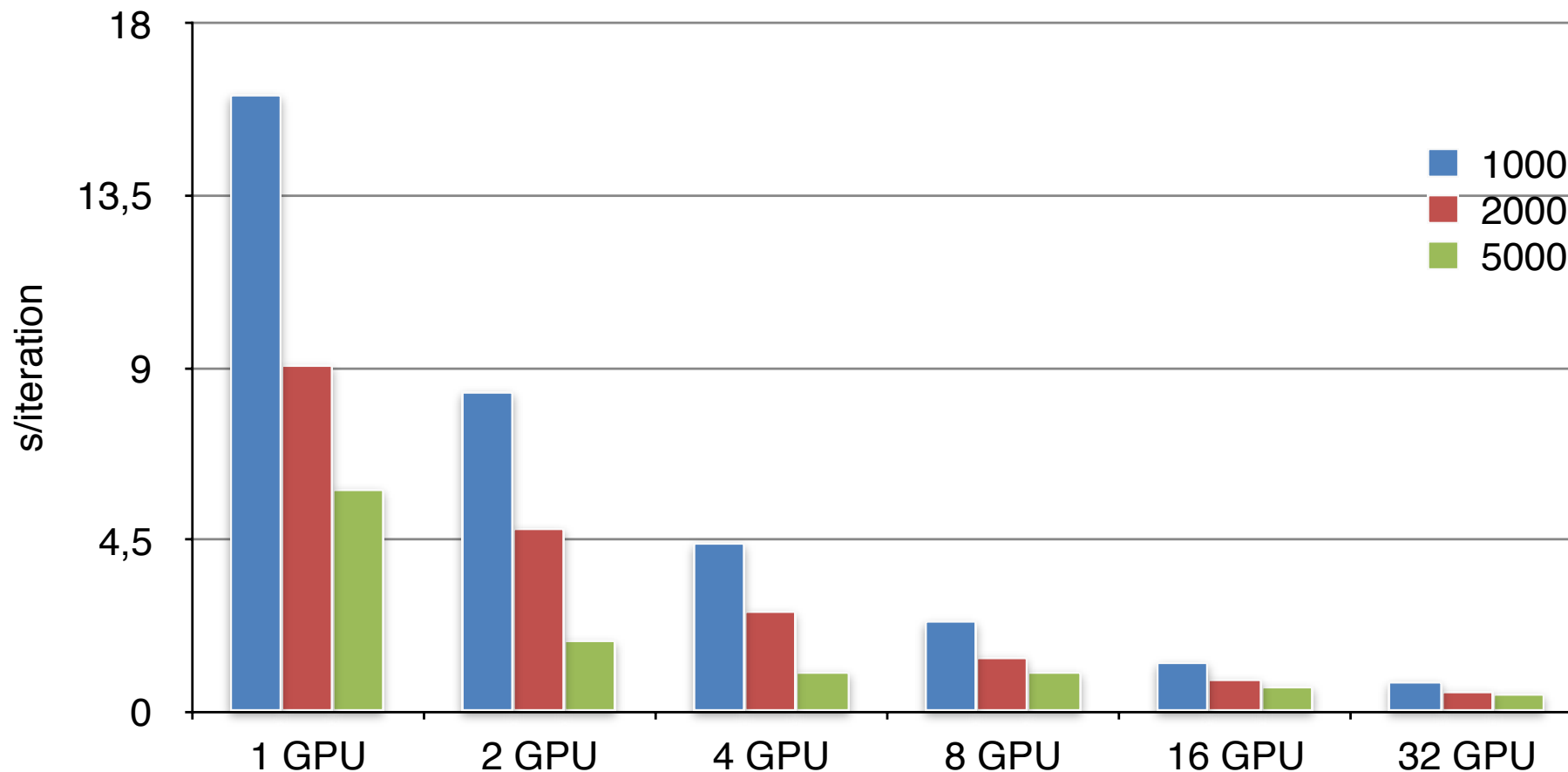


- Scaling: Strong i.e faster result (left) and weak i.e finer mesh (right)



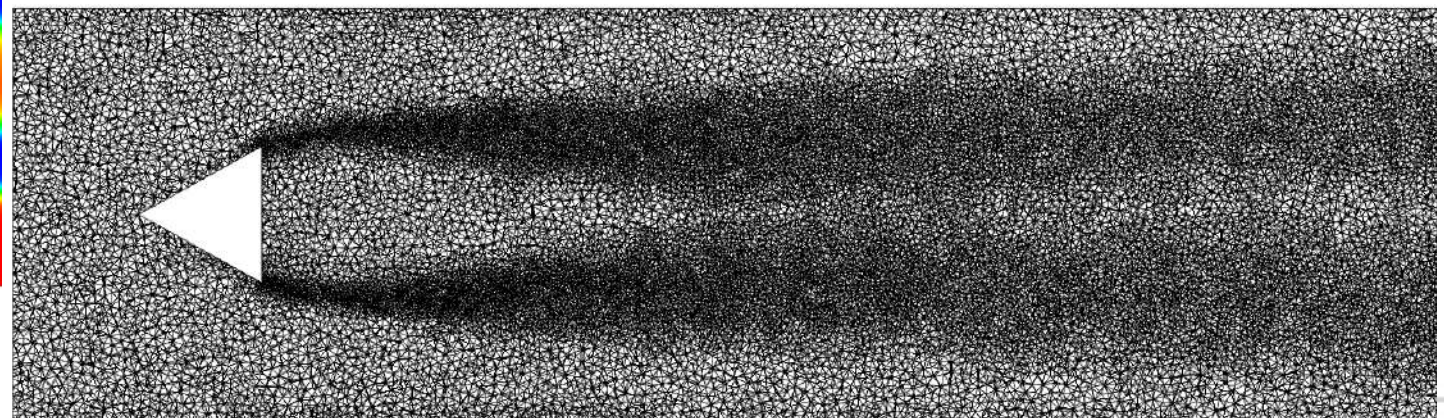
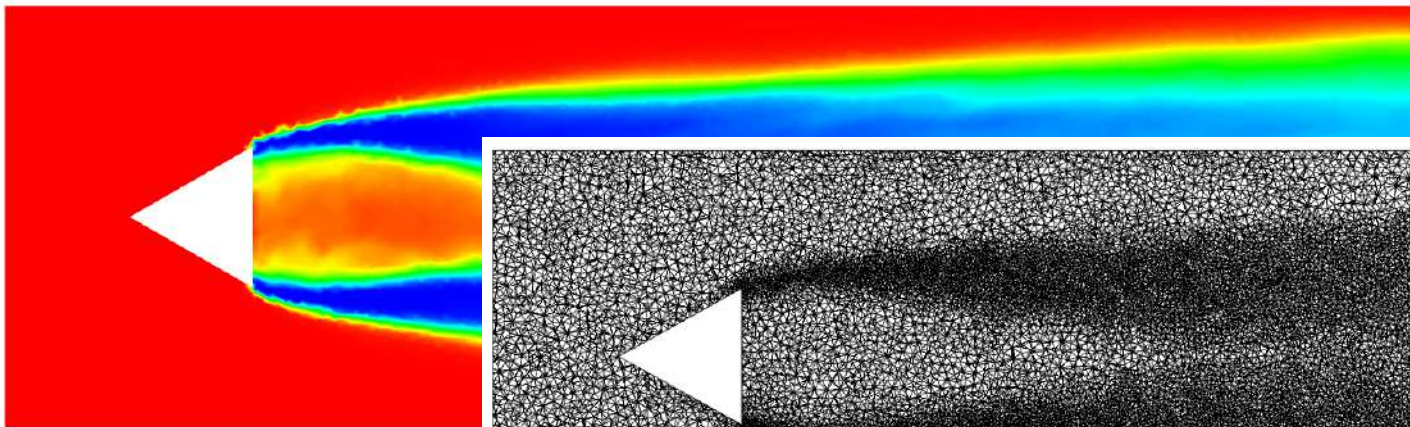
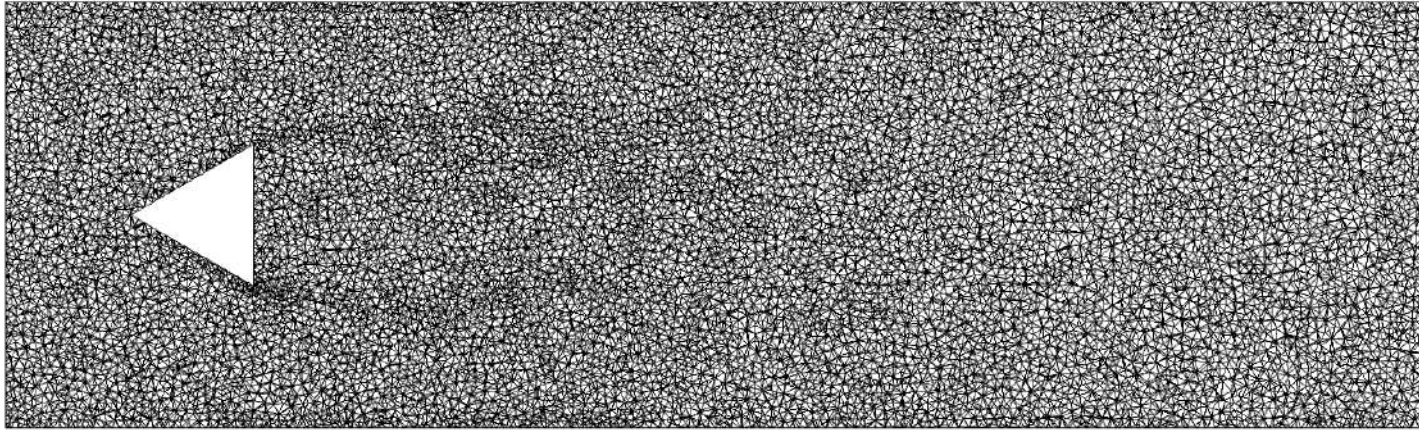


- Excellent strong scaling performance
 - Additional cache optimisations accelerate the simulations
- Influence of the Cell group size





- Adaptation using Grad C/C



Key ingredients for accurate and reliable numerical simulation of turbulent combustion:

- High-quality, adapted mesh
- High order integration schemes
- Parallel efficiency

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