Counterparty Credit Exposure Calculation under IMM

SIAM Student Day

23 May 2016

Contents

Introduction (1/4)

- Big Picture Post-crisis:
  - Regulation reforms:
    - Basel 2.5 (IRC: market risk due to credit deterioration in bond issuers)
    - Basel 3 (higher capital ratio; CVA RC)
    - Fundamental Review of Trading Book (new market risk RC framework: VaR to ES, IRC to IDR, etc.)
    - On-going revision of CCR RC, CVA RC, etc.
  - Single Supervisory Mechanism (since Dec 2014)
    - Direct supervision by ECB instead of DNB
    - Stricter regulation: Asset Quality Review; On-site investigation
  - Regarding pricing:
    - Exotic products are less (more standardized products)
    - Pricing for simple products is more complicated (multi-curve)
    - Short-term interest rates can be negative

Introduction (2/4)

- Topic for today: Credit exposure calculation under IMM (key component for RC CCR and CVA)

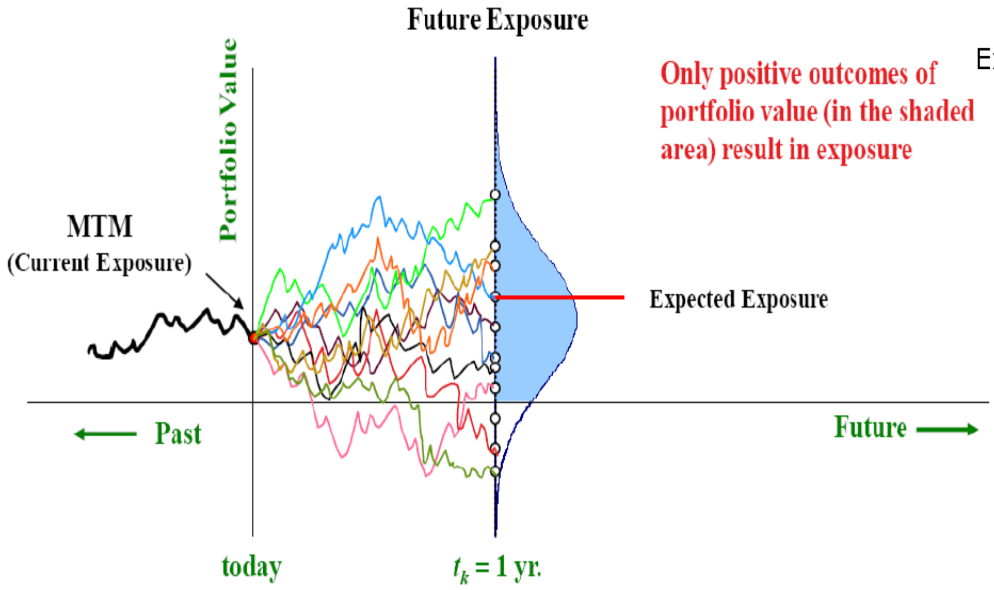
- What is counterparty credit risk (CCR)?
  - The risk that the counterparty will fail to fulfill their side of the agreements

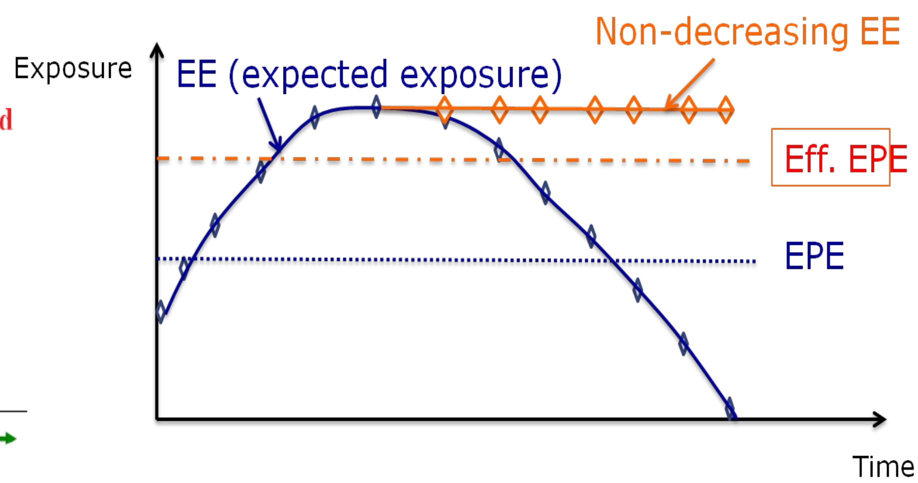
- Difference to credit risk in loans
  - Exposures are not known in advance

- Difference to market risk
  - Market risk: loss in mark-to-market values due to market movements
  - CCR: loss due to default of counterparty

Introduction (3/4)

- Counterparty credit exposure metrics:
  - Expected Exposure (EE)
  - Potential Future Exposure (PFE)
  - Expected Positive Exposure (EPE)
  - Eff. Expected Positive Exposure(EEPE)





Introduction (4/4)

- Means to manage/mitigate CCR
  - Enter collateral agreement: transfer CCR to liquidity risk
  - Set up credit limits in trading activities (needs PFE)
  - Hedging: not always possible
  - Price-in CCR when trading: CVA, DVA (needs EE)

- Regulation requirements:
  - RC for CCR (needs EEPE); Allow own calculation on EE under Internal Model Method (BASEL II)
  - RC for CVA (BASEL III; needs EE)

Counterparty Credit Exposure Calculation (1/2)

Counterparty Credit Exposure Calculation (2/2)

- Main challenges:
  - Selection of risk factor models and calibration of model parameters: main source for model risks
    - Simplest model but complicated enough to capture main properties of the underlying risk factors
    - Calibration of model parameters
  - Pricing: balance between calculation speed and accuracy
    - More assumption thus deviate from front-office price
    - double validation standard

- Risk neutral or real-world?
  - For PFE calculation: P in risk factor simulation, Q in pricing
  - For CVA calculation: Q in both

Example (1/5)

- CMS cap/floor
  - A series of caplets/floorlets, which are call/put options on constant swap rate.
  - Commonly traded options on CMS rates; used to hedge instruments with long maturities

- Needed main risk factors:
  - Yield curve(s)

- Value at time t under forward measure:

- Payoff :

$$V_t = \delta N P_d(t, \tau_0) \mathbb{E}^{T_{d,0}} \left[P_d(\tau_0, T_{\mathrm{p}}) R_i(\tau_0) | \mathcal{F}_t \right],$$

$$R_i(\tau_0) = \left[w \cdot (S_F(\tau_0; \tau_0, \tau_n) - K) \right]^+$$

Example (2/5)

- Forward swap rate:

$$S_F(t;\tau_0,\tau_n) := \frac{\sum_{j=1}^{n} \gamma_j^L P_d(t,\tau_j) F(t;\tau_{j-1},\tau_j)}{\sum_{j=1}^{n} \gamma_j^F P_d(t,\tau_j)}$$

- Forward Libor rate: (assuming Libor rate is martingale under discount-forward measure)

$$F(t;T_{\mathrm{s}},T_{\mathrm{e}}) := \mathbb{E}^{T_{d,\mathrm{e}}}\left[L(T_{\mathrm{s}},T_{\mathrm{e}}) \middle| \mathcal{F}_t \right]$$

- Libor rate:

$$F(T_{\mathrm{s}};T_{\mathrm{s}},T_{\mathrm{e}}) := L(T_{\mathrm{s}},T_{\mathrm{e}})$$

$$L(t,T) := \frac{1 - P_f(t,T)}{\tau P_f(t,T)}$$

Example (3/5)

- Change to annuity measure (because swaption-vols are quoted as such):

$$V_t = \delta N A(t;\tau_0,\tau_n)\mathbb{E}^A\left[\frac{P_d(\tau_0,T_\mathrm{p})R_i(\tau_0)}{A(\tau_0;\tau_0,\tau_n)}\middle|\mathcal{F}_t\right]$$

$$A(t;\tau_0,\tau_n) := \sum_{j=1}^{n}\gamma_j^F P_d(t,\tau_j)$$

- Assuming shifted lognormal for forward swap rate:

$$\begin{aligned}\bar{S}_F(t;T_\mathrm{s},T_\mathrm{e}) &:= S_F(t;T_\mathrm{s},T_\mathrm{e})+\theta\\ d\bar{S}_F(t;T_\mathrm{s},T_\mathrm{e}) &= \bar{S}_F(t;T_\mathrm{s},T_\mathrm{e})\bar{\sigma}\,dW_t\end{aligned}$$

- Rewrite payoff … fted strike:

$$(S_F(\tau_0;\tau_0,\tau_n)-K)^+ = \left(\bar{S}_F(\tau_0;\tau_0,\tau_n)-\bar{K}\right)^+$$

Example (4/5)

- Assume linear swap rate model:

$$\frac{P_d(\tau_0, T_\mathrm{p})}{A(\tau_0; \tau_0, \tau_n)} = \alpha + \bar{\beta}_\mathrm{p} \bar{S}_F(\tau_0; \tau_0, \tau_n)$$

- Inserting this into the expectation:

$$\mathbb{E}^A \left[\frac{P_d(\tau_0, T_\mathrm{p}) R_i(\tau_0)}{A(\tau_0; \tau_0, \tau_n)} \middle| \mathcal{F}_t \right]$$

$$= \mathbb{E}^A \left[\left(\alpha + \bar{\beta}_\mathrm{p} \bar{S}_F(\tau_0; \tau_0, \tau_n) \right) \left(\bar{S}_F(\tau_0; \tau_0, \tau_n) - \bar{K} \right)^+ \middle| \mathcal{F}_t \right]$$

$$= \alpha \cdot \mathsf{Opt}_\mathsf{BS} \left(\bar{S}_F(t; \tau_0, \tau_n), \bar{K}, \bar{\sigma} \right) + \bar{\beta}_\mathrm{p} \bar{S}_F(t; \tau_0, \tau_n) \cdot \mathsf{Opt}_\mathsf{BS} \left(\bar{S}^*_F(t; \tau_0, \tau_n), \bar{K}, \bar{\sigma} \right)$$

$$\bar{S}^*_F(t; \tau_0, \tau_n) := \bar{S}_F(t; \tau_0, \tau_n) \cdot \exp\left(\bar{\sigma}^2 (\tau_0 - t) \right)$$

Example (5/5)

- Calculation steps:
  - Simulate yield curves for a few forecasting dates
  - Loop though all forecasting dates, whereby
    - Loop through all simulated yield curve scenarios, whereby
      - Generate a date strip of the cap/floor and loop though each coupon (caplet/floorlet) period, whereby
        - Generate a data strip of the underlying swap and loop though each Libor period; then aggregate Libor rates according to forward swap rate formula;
        - Apply the CMS caplet/floorlet formula on the swap rate to return the coupon value
      - Sum up discounted coupons to get the MtM value at this forecasting date and for this scenario.

- In the end, we get MtM distributions for all forecasting dates.
- What’s more?(Collateral;netting)

Latest Developments in Regulations

- https://www.bis.org/bcbs/publ/d362.pdf
  - EE from IMM is not allowed for CVA VaR any more
  - There might be a floor to CCR RC based on standardized method