# Student Computational Finance Day 2016

SIAM Student Chapter Delft

May 23, 2016

**Sponsors:**

## Program

The Student Computational Finance Day takes place on May 23rd, 2016 at Technische Universiteit Delft, Faculteit Elektrotechniek, Wiskunde en Informatica. We meet at **Dijkstrazaal HB 09.150**, Mekelweg 4, 2628 CD Delft, The Netherlands.

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| 09:30 - 09:40 | | Welcoming |
| 09:40 - 10:20 | C.W. Oosterlee | Computational finance: numerical techniques and applications |
| 10:40 - 11:00 | Anastasia | Pricing Bermudan options for local Lévy models with default |
| 11:00 - 11:20 | Zaza | The COS method for option valuation under the SABR dynamics |
| 11:20 - 11:40 | Gemma | Computing market risk measures with stochastic holding period by using Shannon wavelet expansions |
| 11:40 - 12:00 | Shih-Hau | Towards Efficient Nonlinear Option Pricing with GPU Computing |
| | | Chairman: Maarten |
| 12:00 - 13:30 | | Lunch at TU Delft Sports Center |
| 13:30 - 14:00 | F. Fang | Calculation of counterparty credit exposures under Internal Model Method |
| 14:10 - 14:30 | Fei | Multi-period Mean-Variance Portfolio Optimization based on Monte-Carlo Simulation |
| 14:30 - 14:50 | Qian | Credit Value Adjustment, Wrong Way Risk and Bermudan Options |
| | | Chairman: Zaza |
| 15:20 - 15:40 | Kees de G. | Efficient CVA computation by risk factor decomposition |
| 15:40 - 16:00 | Slobodan | Radial Basis Functions generated Finite Differences (RBF-FD) for Solving High-Dimensional PDEs in Finance |
| 16:00 - 16:20 | Maarten | ADI Finite Difference Schemes for the Calibration of Stochastic Local Volatility Models |
| | | Chairman: Shih-Hau |
| 16:30 - 17:00 | B.D. Kandhai | Challenges in Computational Finance |
| 17:10 - 18:00 | | Snacks & drinks at TU Delft |

In the evening we will organize a BBQ. Everybody is welcome to join!

# Computational finance: numerical techniques and applications

**Cornelis W. Oosterlee**[*1,2]

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In this presentation, we will explain which numerical and computational techniques are typically used in specific quantitative applications in finance. The techniques will range from partial differential equation solutions to Monte Carlo methods and Fourier techniques.

The applications range from financial product pricing to risk management and portfolio optimization. Each of the numerical methods will be discussed in subsequent presentations during the computational finance day.

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# Pricing Bermudan options for local Lévy models with default

**Anastasia Borovykh**[*1], **Andrea Pascucci**[1] **and**

**Cornelis W. Oosterlee**[2,3]

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We consider a defaultable asset whose risk-neutral pricing dynamics are described by an exponential Lévy-type martingale subject to default. This class of models allows for local volatility, local default intensity and a locally dependent Lévy measure. We present a pricing method for Bermudan options based on an analytical approximation of the characteristic function combined with the COS method. Due to a special form of the characteristic function the price can be computed using a Fast Fourier Transform-based algorithm resulting in a fast and accurate calculation. The Greeks can be computed at almost no additional computational cost. Error bounds for the approximation of the characteristic function as well as for the total option price are given.

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# The COS method for option valuation under the SABR dynamics

Zaza van der Have*[1,2] and Cornelis W. Oosterlee[1,2]

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Efficient valuation of financial derivatives is an important issue in financial mathematics. Fourier methods, such as the COS method [1, 2], employ the characteristic function of the underlying asset process to determine the option value, where a characteristic function is the Fourier transform of the underlying density. However, often no analytic expression for the characteristic function of the underlying process is available, like for the SABR model.

This model, also known as the "stochastic alpha beta rho model", is since the introduction of the Hagan formula [3] in 2002 a widely used stochastic volatility model. We propose to use the bivariate characteristic function of the discretized SABR process to price European and Bermudan options, where we use the Euler-Maruyama or the 2.0-weak-Taylor scheme for the discretization. The application of these schemes in combination with the COS method results respectively in first-order and second-order convergence.

Second-order convergence can also be obtained by using Richardson extrapolation in combination with an Euler-Maruyama discretization on the forward process, which provides a significant reduction in computational costs compared to the 2.0-weak-Taylor scheme.

We also solve backward stochastic differential equations by using the discretized stochastic processes and the Fourier-cosine expansion. For this purpose we use the BCOS method (Backward Stochastic Differential Equation COS method) [4], which we extended from one to two dimensions.

## References

[1] F. Fang and C.W. Oosterlee, A Novel Pricing Method for European Options Based on Fourier-Cosine Series Expansions, *SIAM Journal on Scientific Computing*, **31**, pp. 826-848, 2008.

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[2] M.J. Ruijter and C.W. Oosterlee, Two-dimensional Fourier Cosine Series Expansion Method for Pricing Financial Options, *SIAM Journal on Scientific Computing*, **34**(5), pp. B642-B671, 2012.

[3] P.S. Hagan, D. Kumar, A.S. Lesniewski, and D.E. Woodward, Managing Smile Risk, *Wilmott Magazine*, pp. 84-108, 2002.

[4] M.J. Ruijter and C.W. Oosterlee, Numerical Fourier Method and second-order Taylor Scheme for Backward SDEs in Finance, *To be published in Applied Numerical Mathematics*, 2016.

# Computing market risk measures with stochastic holding period by using Shannon wavelet expansions

**Gemma Colldeforns-Papiol**[*1,2], **Luis Ortiz-Gracia**[1,2] **and Cornelis W. Oosterlee**[3,4]

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The Basel Committee of Banking Supervision (BCBS) states in recent consultative documents that the financial crisis exposed material weaknesses in the overall design of the framework for capitalising trading activities. Due to that, the BCBS has focused on incorporating the risk of market illiquidity as a key consideration in banks' regulatory capital requirements for trading portfolios. Motivated by this, our purpose is to present a set of numerical techniques to compute the Value-at-Risk (VaR) and expected shortfall (ES) risk measures under a stochastic time horizon to take into account the market liquidity. This idea was first introduced by [1] and we give a step further by considering different dynamics to drive the portfolio.

To evaluate risk measures, Monte Carlo simulation is often used, but obtaining accurate estimates is computationally expensive. Here we adopt scenarios where the characteristic function of a fixed portfolio change is known in closed form. We provide a procedure to express the characteristic function of a stochastic time horizon portfolio change from the deterministic one. We provide methodology for computing the VaR and ES using SWIFT [2], which is a technique based on a Shannon wavelet expansion of a density function assuming its Fourier transform is known. Shannon wavelets are smooth wavelets based on the cardinal sine function. From its nature SWIFT presents several benefits such as high accuracy, robustness, fast convergence, the density approximation does not deteriorate with the choice of the domain size, the number of terms needed are automatically calculated and estimation of the error is available.

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# References


[1] D. Brigo, C. Nordio, A random holding period approach for liquidity-inclusive risk management, In K. Glau, (ed.), *Innovations in Quantitative Risk Management* **47**, Springer Proceedings in Mathematics & Statistics, 2015.

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# Towards Efficient Nonlinear Option Pricing with GPU Computing

**Shih-Hau Tan**[*1], **Kevin Parrott**[1] **and Choi-Hong Lai**[1]

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Nonlinear option pricing is a new approach for traders, hedge funds or banks to obtain more accurate option price and allow them to do fast model calibration using huge market data. The idea is to take into account nontrivial transaction costs, liquidity, market feedback and risk from unprotected portfolio to the pricing model ([1, 2]). Numerically the main problem is to solve fully nonlinear PDEs which are generalized Black-Scholes equation with nonlinear volatility, and strategy like Newton's method is employed to handle the nonlinearity.

In this presentation, we aim to introduce the work on the importance of these nonlinear models in financial market, and efficient solvers to do the computation. The purpose is to solve the large-scale nonlinear option pricing problems by using GPUs with batch operation ([3]). Essentially the implementation is to do computation of some level-2 functions with special designed kernel functions for tri-diagonal matrix, and also to apply parallel tri-diagonal solver to deal with the linear system. We will show the comparison of speedups by implementing with OpenACC, CUDA libraries and kernel functions, and also some results by using multiple GPUs.

## References

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[3] M.B. Giles, E. Laszlo, I. Reguly, J. Appleyard, J. Demouth, GPU implementation of finite difference solvers, Seventh Workshop on High Performance Computational Finance (WHPCF'14), IEEE, 2014.

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# Calculation of Counterparty Credit Exposures under Internal Model Method

**Fang Fang***[1]

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Counterparty credit risk (CCR) arises from the credit risk in securities financing transactions such as repos and Over The Counter (OTC) derivatives market. It is the risk that a counterparty will default on a transaction prior to the expiration of the contract and will be unable to make all contractual payments. The credit exposure in case of a default at any future date is the replacement cost of the derivative, which is determined by the market value at that date which is uncertain. Under the IMM method as defined by BASEL II, credit exposures are calculated based on Monte Carlo simulation scenarios of important risk factors. In this presentation I will use an example to illustrate how the calculation is done, what are the difficulties in MtM pricing, how the results are used in a bank, and what are the latest developments in the regulation.

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# Multi-period Mean-Variance Portfolio Optimization based on Monte-Carlo Simulation

**Fei Cong***[1] **and Cornelis W. Oosterlee**[1,2]

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We propose a simulation-based approach for solving the constrained dynamic mean-variance portfolio management problem. For this dynamic optimization problem, we first consider a sub-optimal strategy, called the multi-stage strategy, which can be utilized in a forward fashion. Then, based on this fast yet sub-optimal strategy, we propose a backward recursive programming approach to improve it. We design the backward recursion algorithm such that the result is guaranteed to converge to a solution, which is at least as good as the one generated by the multi-stage strategy. In our numerical tests, highly satisfactory asset allocations are obtained for dynamic portfolio management problems with realistic constraints on the control variables.

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# Credit Value Adjustment, Wrong Way Risk and Bermudan Options

**Qian Feng**[*1] **and Cornelis W. Oosterlee**[1,2]

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Credit value adjustment (CVA) is an capital charge introduced in Basel III to improve the bank's resilience against future loss due to the default of the counterparty. The complexity of computing CVA arises from the uncertainties of the losses of a default event (exposure), the likelihood of the counterpartys default in the future (default probability), and the implicit dependence structure between exposure and default probability. When the counterparty's credit quality and the associated exposure is adversely correlated, the effect called wrong way risk (WWR) incurs. Banks are required to identify and monitor WWR in Basel III as this effect may make significant contribution to CVA.

Bermudan option holders have the right to exercise the options in a set of time steps, and thus may change the early exercise strategy when they want to reduce the counterparty credit risk (CCR). Banks will be overcharged if they compute CVA of Bermudan options without considering the changed behaviors of the option investors in a default circumstance.

This paper proposes an affine model to describe the dependence structure between the underlying equity and the default probability of the counterparty based on intensity approach. By defining concepts of default-free and default-adjusted values, we present that CVA values that accounts WWR or RWR in the future time steps can be computed without nest Monte Carlo simulation under this framework, which enhances the computational efficiency greatly.

We show that the Monte Carlo method SGBM (Stochastic Grid Bundling Method) and the Fourier cosine expansion method COS can both be applied efficiently to price Bermudan options in this context. We analyze the impact of CCR and WWR on Bermudan options. We compare the optimal early exercise values obtained with two types of exercise principles: one is aimed to maximize the default-free value, and the other is to maximize the default-adjusted value.

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The results show that a risk-averse option holder will exercise the option earlier when there is more likelihood of a default event. We further see that the CVA of a Bermudan option is smaller but not eliminated when it is exercised by maximizing the default-adjusted value. The optimal early exercise values are also different when WWR or RWR incurs.

# Efficient CVA computation by risk factor decomposition

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According to Basel III, financial institutions have to charge a Credit Valuation Adjustment (CVA) to account for a possible counterparty default. Calculating this measure is one of the big challenges in risk management. In earlier studies, future distributions of derivative values have been simulated by a combination of finite difference methods for the option valuation and Monte Carlo methods for the state space sampling of the underlying, from which the portfolio exposure and its quantiles can be estimated.

By solving a forward Kolmogorov PDE for the future underlying distribution instead of Monte Carlo simulation, we hope to achieve efficiency gains and better accuracy especially in the tails of future exposures. Together with the backward Kolmogorov equation, the expected exposure and quantiles can then directly be obtained without the need for an extra Monte Carlo simulation. We studied the applicability of PCA and ANOVA-based dimension reduction in the context of a portfolio of risk factors. Typically, for these portfolios, a huge number of derivatives are traded on a relatively small number of risk factors. By solving a PDE for one risk factor, it is possible to value all derivatives traded on this single factor over time. However, if we want to solve a PDE for multiple risk factors, one has to deal with the curse of dimensionality. Between these risk factors, the correlation is often high, and therefore PCA and ANOVA are promising techniques for dimension reduction and can enable us to compute the exposure profiles for higher dimensional portfolios.

We compute lower dimensional approximations where only one factor is taken stochastic and all other factors follow a deterministic term structure. Next, we correct this low dimensional approximation by two dimensional approximations. We also look into the effect of taking higher (three) dimensional

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corrections.

In our results, our method is able to compute Exposures (EE, EPE and ENE) and Quantiles for a real portfolio driven by 7 different risk factors. This portfolio consists of Cross-Currency Swaps, Interest rate swaps and FX call or put options. The risk factors are: stochastic FX rates and stochastic domestic and foreign interest rates. The method is accurate and fast when compared to a full-scale Monte Carlo implementation.

# Radial Basis Functions generated Finite Differences (RBF-FD) for Solving High-Dimensional PDEs in Finance

**Slobodan Milovanović**[*1] and **Lina von Sydow**[1]

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A radial basis function generated finite difference (RBF-FD) method has been considered for solving PDEs arising in pricing of financial contracts. By being mesh-free while yielding a sparse differentiation matrix, this method aims to exploit the best properties from, both, finite difference (FD) methods and radial basis function (RBF) methods. Moreover, the RBF-FD method is expected to be advantageous for high-dimensional problems compared to: Monte Carlo (MC) methods which converge slowly, global RBF methods since they produce dense matrices, and FD methods because they require regular grids. Implementations have been done for the standard Black-Scholes-Merton equation to price European and American options with discrete or continuous dividends in 1D, and European call basket and spread options in 2D on adapted domains. Performance of the method and the error profiles have been studied with respect to discretization in space, size and form of stencils, RBF shape parameter and boundary conditions. The results highlight RBF-FD as a competitive, sparse method, capable of achieving high accuracy with a small number of nodes in space.

## References

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[4] L. von Sydow, L. J. Höök, E. Larsson, E. Lindström, S. Milovanović, J. Persson, V. Shcherbakov, Y. Shpolyanskiy, S. Sirén, J. Toivanen, J. Waldén, M. Wiktorsson, M. B. Giles, J. Levesley, J. Li, C. W. Oosterlee, M. J. Ruijter, A. Toropov, Y. Zhao, *BENCHOP  The BENCHmarking project in Option Pricing*, International Journal of Computer Mathematics, Vol. 92, pg. 2361–2379, 2015.

# ADI Finite Difference Schemes for the Calibration of Stochastic Local Volatility Models

**Maarten Wyns**[*1] **and Karel J. in 't Hout**[1]

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In contemporary financial mathematics, stochastic local volatility (SLV) models form state-of-the-art models to describe asset price processes. The local component of the SLV model, the so-called leverage function, is defined in a natural way such that the SLV model yields the same fair value for vanilla options as the underlying local volatility (LV) model. Determining this leverage function is, however, a highly non-trivial task. For example, the fair option values defined by the LV model can often not be obtained analytically and have to be approximated, e.g. by numerically solving the corresponding backward PDE.

Consider any given discretization by finite differences of the one-dimensional backward PDE from the LV model and suppose discretization of the two-dimensional backward PDE corresponding with the SLV model is performed by similar finite difference formulas. In this talk we shall propose a calibration technique with the useful property that it determines the leverage function such that both discretizations define exactly the same approximation for the fair value of vanilla options. In this calibration procedure, which involves a two-dimensional PDE problem, alternating direction implicit (ADI) time stepping schemes are used as they are highly efficient in comparison to classical implicit methods. Ample numerical experiments are provided that illustrate the effectiveness of this calibration procedure.

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# Challenges in Computational Finance

**B.D. (Drona) Kandhai***[1,2]

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In this talk I will guide you through the world of financial derivatives, their valuation and risk management and, more importantly, how the recent credit crisis has completely changed the landscape. The impact of this change on the modeling and computational complexity and the related challenges will be discussed. A couple of recent research projects in close collaboration with industrial and other academic partners will be highlighted.

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