Credit Value Adjustment, Bermudan option and Wrong Way Risk

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1. Counterparty Credit Risk
2. Credit Value Adjustment
3. Bermudan option
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Counterparty Credit Risk

Counterparty Credit Risk (CCR)

CCR is the risk to each party of a contract that the counterparty may fail to fulfill its obligations, causing losses to the other party.

Example

Company A agrees to lend Company B a certain amount of money. It is expected that company A will provide the money on time and company B will pay the money back. There is counterparty risk for both that company B may not be able to pay the loan while company A may stop providing the agreed upon funds.

OTC (over-the-counter) derivative contract

- Interest rate swap,
- FX forwards
- ...

(CWI)
Basel Committee on Banking Supervision

The Basel Committee on Banking Supervision (BCBS) is a committee of banking supervisory authorities aiming to improve the quality of banking supervision worldwide.

Credit exposure

the replacement cost of the contract, which is equal to the greater of the fair market value of the contract and zero.

Exposure measurements

- Basel II: Expected Exposure (EE), Potential future exposure (PFE)
Credit Value Adjustment (CVA)

CVA is the market value of counterparty credit risk:

\[
CVA = \mathbb{E}^Q \left[ \text{LGD} \int_0^T D(0, t) E_t dPD(t) \right],
\]

(1)

where

- **LGD**: the percentage of loss given default;
- **E**: exposure \( E_t = \max(V_t, 0) \), where \( V_t \) represents the mark-to-market value of the portfolio;
- **PD**: the default probability, \( dPD(t) = PS(t + dt) - PS(t) \) the probability that default happens during time \([t, t + dt]\);
- **Q**: risk-neutral probability measure.
- **D**: the discounting factor \( D(0, t) = 1/B(t) \), where banking account \( B(t) = \exp\left(-\int_0^t r_s ds\right) \) with the risk-free short rate \( r_s \);
Intensity approach

Survival probability

\[ G(s, t) = \exp \left( - \int_s^t h_u \, du \right) , \] (2)

where \( h_u \) represents the positive intensity at time \( u \):

- \( h_u \, du \) defines the probability that default time occurs during period \([u, u + du]\).
- \( h_u \) can be a constant, a deterministic function, a stochastic process...

Marginal default/survival probability model

Survival: \( PS(t) = \mathbb{E}_Q [G(0, t)] \). (3)

Default: \( PD(t) = 1 - PS(t) \). (4)
Bermudan option

A Bermudan option is an option that be exercised on a number of dates.

Payoff

Let $S_t$ be the stock price at time $t$

$$g(S_t) = \begin{cases} \max(0, K - S_t) & \text{put}, \\ \max(0, S_t - K) & \text{call}, \end{cases}$$

(5)

where $K$ the fixed strike.

Exercise dates

Let $T = \{0 < t_1 < \ldots < t_N = T\}$ be the collection of early-exercise dates.
Bermudan option II

Default-free value

\[ V_0 = \max_{\tau \in T} \mathbb{E}^Q\left[ D(0, \tau) g(S_{\tau}) \right], \]  
(6)

where \( \tau \) is the exercise time, and let \( \tau^* \) be the solution which represents the optimal exercise time when maximizing the default-free value.

Default-adjusted value

\[ A_0 = \max_{\beta \in T} \mathbb{E}^Q\left[ D(0, \beta) G(0, \beta) g(S_{\beta}) \right]. \]  
(7)

where \( \beta \) is the exercise time, and let \( \beta^* \) be the solution which represents the optimal exercise time when maximizing the default-adjusted value.
Optimal exercise boundary at time $t_m \in \mathcal{T}$

Optimal default-free exercise value $x^*(t_m)$:

$$V_t(x^*(t_m)) - g(x^*(t_m)) = 0,$$  \hspace{1cm} (8)

where the option value at time $t_m$ conditioned on $S_{t_m} = x$ is given by

$$V_t(x) = \max_{\tau \in \{t_m+1, \ldots, t_M\}} \mathbb{E}^\mathbb{Q}\left[D(t, \tau)g(S_{\tau}) \mid S_{t_m} = x\right].$$ \hspace{1cm} (9)

Optimal default-adjusted exercise value $y^*(t_m)$:

$$A_t(y^*(t_m)) - g(y^*(t_m)) = 0,$$ \hspace{1cm} (10)

where the option value at time $t_m$ conditioned on $S_{t_m} = x$ is given by

$$A_t(x) = \max_{\beta \in \{t_m+1, \ldots, t_M\}} \mathbb{E}^\mathbb{Q}\left[D(t, \beta)G(t, \beta)g(S_{\beta}) \mid S_{t_m} = x\right].$$ \hspace{1cm} (11)
Numerical Schemes

COS method
Fourier-cosine expansion and FFT

- Fang and Oosterlee, Ruijter and Oosterlee ...

Stochastic Grid Bundling Method (SGBM)
Simulation, regression, bundling, and the relation of the (discounted) characteristic function and the (discounted) moments.

- Jain and Oosterlee ...
Optimal early exercise boundary

Optimal early exercise values of a Bermudan put option with constant intensity $h = \{0.03, 0.3\}$ over period $[0, T]$. Parameters $S_0 = 100$, $r = 0.01$, $\sigma = 0.4$. Expiration $T = 0.25$, early exercise steps $M = 10$, strike $K = 100$. 
**Bermudan options**

\[ \beta^* : \text{exercised by maximizing default-adjusted value} \]

<table>
<thead>
<tr>
<th>Intensity</th>
<th>default-adjusted value</th>
<th>default-free value</th>
<th>CVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h = 0.03 )</td>
<td>7.7943</td>
<td>7.8416</td>
<td>0.0473</td>
</tr>
<tr>
<td>( h = 0.3 )</td>
<td>7.4077</td>
<td>7.8162</td>
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\[ \tau^* : \text{exercised by maximizing default-free value} \]

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<tbody>
<tr>
<td>( h = 0.03 )</td>
<td>7.7936</td>
<td>7.8422</td>
<td>0.0486</td>
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<tr>
<td>( h = 0.3 )</td>
<td>7.3700</td>
<td>7.8422</td>
<td>0.4722</td>
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</tbody>
</table>
Wrong Way Risk (WWR)

This type of risk occurs when exposure to a counterparty is adversely correlated with the credit quality of that counterparty.

Example

A put option written by bank A on equity of bank B, and bank A and bank B have similar portfolio:

- Credit quality of bank A and bank B goes worse;
- stock price of bank B decreases;
- market value of the put option increases;
- the likelihood of default of bank A increases;
Hull-White model

\[ dx_t = \left( r - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t^1, \]  
\[ dy_t = \gamma(\bar{y} - y_t) dt + \eta dW_t^2, \]  
\[ h_t = \psi(t) + y_t, \]

where \( x_t = \log(S_t) \) is the log-stock value, \( r \) represents the risk-free short rate, \( \sigma > 0 \) the constant implied volatility; \( \gamma > 0 \) corresponds to reversion speed, \( \bar{y} \) to long run reverting level and \( \eta > 0 \) to the volatility of variable \( y_t \), and \( dW_t^1 \) and \( dW_t^2 \) are two Wiener processes with correlation \( dW_t^1 \cdot dW_t^2 = \rho dt \), and \( \rho \) the correlation coefficient; deterministic function \( \psi(t) \) satisfies \( \psi(0) = h_0 - y_0 \).

Drawback: intensity may become negative.
CVA stress testing of w.r.t correlation. Parameters $S_0 = 100$, $r = 0.01$, $\sigma = 0.4$, $\gamma = 0.2$, $\bar{y} = 0.1$, $\eta = 0.2$; marginal survival function $\exp(-0.3t)$ and $y_0 = 0.3$. An European put option with expiration $T = 0.25$ and strike $K = 100$. Worst WWR ratio 1.18; best RWR ratio 0.82.
Conclusion

- Credit risk may change the early exercise strategy for credit-risk-alert investors: the optimal early exercise values are increased for put options and decreased for call options;
- Credit risk and WWR cannot be eliminated by changing the exercise strategy.

Future

- Wrong way risk
Bibliography


